IV. *A New Apparatus for Determining the Relationship between Wave-lengths of Light and the Fundamental Standards of Length.*

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(*Communicated by Sir* JOSEPH PETAVEL, *F.R.S.*).

(Received March 11, 1932.)

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[Published September 16, 1932.

1. *Introduction.*

The idea of establishing the fundamental units of length on a basis defined by some natural standard has long attracted physicists. The metre was originally intended to represent one ten-millionth of a meridional quadrant of the earth,* and the Weights and Measures Act of 1824 prescribed that the yard, if ever lost or destroyed, should be replaced by reference to the length of a pendulum beating seconds in vacuum at sea level in London.† It is believed that BABINET, in 1829, suggested the use of a wave-length of light to define the fundamental unit of length.

The first specific proposal to correlate the wave-length of light with material standards of length was made about 1875 by GOULD,‡ who suggested the use of a diffraction grating for the determination. It was not until 1893 that the first direct measurement of the metre in terms of the red radiation of cadmium was made by MICHELSON and BENOÎT.§ The comparison was made possible by a slightly modified form of the famous interferometer which MICHELSON had devised originally for the attempt to measure the relative velocity of the earth through the ether. In 1906, BENOÎT, FABRY and PEROT|| repeated the determination of the length of the metre in terms of the red radiation of cadmium, using different methods and apparatus. More recently, in 1928, WATANABE and IMAIZUMI¶ have again repeated the determination with apparatus essentially the same as that used by BENOÎT, FABRY and PEROT.

Shortly after BENOÎT, FABRY and PEROT had announced the result of their work, astronomers and spectroscopists, at a meeting of the International Solar Union (now the International Astronomical Union) held in 1907,** adopted the wave-length of the red line of cadmium as the basis of reference for all measurements of wave-lengths of light, and defined its value in terms of the International Ångström. Making use of the relationship between the wave-length of the red line of cadmium and the metre determined by BENOÎT, FABRY and PEROT, which was in close accordance with the previous determination by MICHELSON and BENOÎT, the International Ångström was defined in such a way that it can differ but little from 10^{-10} metre.

In 1923, the International Committee of Weights and Measures accepted in principle the eventual use of a wave-length of light as the ultimate standard of length, subject to the formulation of satisfactory conditions for the practical realisation of such a standard,†† and urged all the national laboratories to initiate researches for the attain-

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* GLAZEBROOK, 'Proc. Phys. Soc.,' vol. 43, p. 450 (1931).

† *Ibid.*, p. 455.

‡ MICHELSON, "Light Waves and Their Uses," p. 84 (1907 edition).

§ 'Trav. Bur. int. Pds. Mes.,' vol. 11, p. 3 (1895).

|| 'Trav. Bur. int. Pds. Mes.,' vol. 15, p. 3 (1913).

¶ 'Proc. Imp. Acad. Japan,' vol. 4, p. 350 (1928).

** 'Trans. Int. Union Solar Res.,' pt. 2, vol. 20, p. 28 (1907).

†† "Comité Internat. Pds. Mes., Procès-Verbaux des Séances," p. 67 (1923).

ment of this end. In 1927 the International Conference on Weights and Measures gave formal sanction,* as an interim measure, to measurements of length being made by reference to the wave-length of the red radiation of cadmium determined by Benoît, Fabry and Perot, as an alternative to direct reference to the metre.

Meanwhile, as a result of the last decennial comparisons of the Imperial Standard Yard with its legal copies at the Board of Trade in 1922, the need had been recognised† of giving early consideration to the problem of determining the best procedure to be adopted for replacing the present Imperial Standard Yard by a standard more in accordance with modern requirements, and the possibility had been suggested of using the wave-length of light in this connection.

With this in view, and having regard also to the resolution of the International Committee in 1923, it was decided, with the approval of the Board of Trade, that the National Physical Laboratory should undertake an investigation having for its objects, firstly, the establishment of suitable means of realising a wave-length standard of length, and, secondly, an accurate determination of the length of the Imperial Standard Yard in terms of the chosen wave-length. The present paper contains a description of the apparatus and methods which have been developed for this purpose, together with a brief account of some preliminary experiments made in the course of testing it which may be regarded as constituting a provisional re-determination of the length of the metre in terms of the wave-length of the cadmium red radiation. It represents, in effect, the completion of the first part of the above programme.

Before entering into a full discussion of the new apparatus it is important to direct attention to another development of historical interest. The Exchequer yard of Queen Elizabeth‡ as also the original Mètre des Archives were end-standards. The superior precision offered at the date of their construction by standards bearing fine graduation lines which could be observed through microscopes, led to the present Imperial Standard Yard and International Prototype Metre being made in the form of line-standards. Early in the present century Johansson first introduced short end-standards or block-gauges made of hardened steel, having terminal faces approaching optical quality in regard to flatness and parallelism. These were considerably improved in succeeding years, and gauges of this type are now accepted as the most convenient and accurate form of length standard in modern high precision workshops. In 1920 the National Physical Laboratory was successful in producing cylindrical rod gauges of considerable length with flat and parallel terminal surfaces of optical perfection.§ The accurate finish of these gauges not only affords a precision in the ascertainment of length which is definitely superior to that provided by line-standards, but at the same

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* "Comité Internat. Pds. Mes., Procès-Verbaux des Séances," p. 67 (1927).

† "Report by the Board of Trade on the Comparisons of the Parliamentary Copies of the Imperial Standards" (1930), p. 11 (H.M. Stationery Office).

‡ Glazebrook, *loc. cit.*, p. 422.

§ "Annual Report of the National Physical Laboratory for 1921," p. 135.

time overcomes the previous objection to end-standards that the terminal faces were liable to damage by contact in measurement, since the new end-standards can be compared by purely optical means, without any contact being made at their surfaces. The obvious advantage of this type of end-gauge as the material representation of the fundamental units of length for direct comparison with the wave-length of light naturally led to its adoption for use in connection with the new apparatus. The methods employed in producing these standards were also applied with equal success to the construction of the auxiliary étalons which constitute an essential feature of the apparatus.

At present the standard values of wave-lengths are those determined in air under specified conditions of temperature, pressure, etc., and corrections must be applied to measurements made under any other conditions in order to compensate for the departure from standard conditions. Therefore measurements of the highest precision involve parallel observations of certain variable factors such as temperature, barometric pressure, humidity and possibly the carbonic acid gas content of the air, the influence of which on the refractive index is calculated by accepting values for certain constants involved in the reduction. But there are disagreements among the published values of these constants, and at the 1931 meeting of the International Committee of Weights and Measures it was agreed that further precise determinations of the refractive index of air were required.* Naturally these determinations must include a complete examination of the influence of the variable factors on the refractive index. Assuming that suitable agreement were found between independent observers, a question would then still arise as to the most suitable method of dealing with the influence of refractive index in connection with any final proposal for the definition of the units of length in terms of the wave-length of light. The new apparatus is so designed that measurements may be made either in air under controlled conditions or in vacuum. It therefore affords ready means for making the new determinations of refractive index which will in any case be needed before a wave-length definition of the unit of length can be finally adopted, since practical every-day measurements cannot conveniently be made in vacuum, and optical measurements of length are already beginning to enter into routine work even in industrial establishments. It also affords means for investigating the possibility of defining the ultimate standard of length by reference to wave-lengths in vacuum and so eliminating atmospheric corrections entirely from the fundamental definition.

The wave-length in vacuum is obviously ideal as a definition because it is easily reproducible under modern conditions of vacuum technique, and because it is, in fact, a truly natural constant. But it may be thought that it might lead to difficulties when applied to the interpretation of direct measurements of lengths which, for practical purposes, may necessarily have to be made in air. A striking example of an apparently similar kind is the definition of the Prototype Metre at the temperature of 0° C., which

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* "Comité Internat. Pds. Mes., Procès-Verbaux des Séances de 1931," p. 47.

was accepted because of its ideal characteristic of reproducibility before the difficulties inherent in making practical every-day measurements at this temperature were sufficiently realised. In the present authors' opinion the parallel is not exact, and the advantage appears to them to lie with a definition in vacuum. Full discussion of this question, however, lies outside the scope of the present paper.

Wave-length determinations of the metre have hitherto all been made in terms of the red radiation of cadmium, which was chosen by MICHELSON as the most suitable for the purpose among the many radiations that he examined. It is not yet definitely established, perhaps, that the red radiation of cadmium as produced by a lamp of the Michelson type provides the most ideal radiation for an ultimate reference standard, and at least one other radiation, the yellow-green line of krypton, has been suggested as an alternative.* This krypton line is undoubtedly very monochromatic, but it is also of low intensity, and has not met with great approval, among spectroscopists in particular, since it lies in the most difficult region, photographically, of the visible spectrum. In the present state of our knowledge, it can be stated that the red ray of cadmium still remains unsurpassed, and that the most satisfactory monochromatic radiation for metrological purposes may yet be found by the evolution of a method of excitation of the cadmium radiation which does not suffer from certain disadvantages associated with the Michelson type of lamp.

2. *Method.*

(*a*) *Basic Measurement of First Étalon.*—The method adopted in the present work for comparing the wave-length of light with a material standard of length is based largely on that used by BENOÎT, FABRY and PEROT, *loc. cit.*, with some modifications.

The initial step in the procedure is the optical determination of the number of waves of some known radiation contained in the distance between the semi-reflecting surfaces of a Fabry-Perot étalon. The Fabry-Perot étalon consists of a pair of mutually parallel semi-silvered glass or quartz plates, held at a fixed distance apart, and interferential methods are well established whereby this distance may be measured in terms of waves of light. There is, however, a limit to the mirror separation which can be standardised by these methods, imposed by the fact that complete disappearance of interference phenomena takes place when the mirrors are separated by more than a certain distance. This is brought about because, owing to the Doppler effect, if for no other reason, ideally homogeneous radiation is unobtainable. Accordingly, the initial measurement in the BENOÎT, FABRY and PEROT determination was applied to an étalon of approximately 1/16-metre separation; but in the present work the étalon separation or length is increased to about 1/12-metre, thereby leading to a greater relative accuracy in the basic measurement. It is considered that the slightly decreased

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* "Comité Internat. Pds. Mes., Procès-Verbaux des Séances," p. 68 (1927).

visibility of the interference phenomena due to the longer étalon is not accompanied by an equivalent decrease in the accuracy of measurement. An alternative étalon of about 1/9-metre is provided so that the final result may be derived from two independent basic measurements.

The type of étalon used by BENOÎT, FABRY and PEROT consisted of an invar bar of channel section with three studs at each end against which the semi-silvered glass plates were held by springs, and was provided with means for adjusting the semi-silvered surfaces to parallelism. In the present apparatus the étalon is of tubular form, also made of invar, and the ends of the tube are chromium plated. Optically flat and parallel surfaces are produced on these hard plated ends, and flat glass or quartz plates are brought into direct wringing contact with these surfaces. Before the plates are wrung to the tube a thin partially reflecting silver film is deposited, by the method of cathodic sputtering in vacuum, on that portion of each plate which covers the bore of the tube. Adjustments for parallelism and also, to a limited extent, for length, are provided by means of straining wires or rods of invar joining two flanges on the external surface of the tube. The étalon so constituted is exceptionally rigid and is capable of withstanding incidental vibration and shock without affecting its adjustment in any way. Another important feature is that as the terminal plates form air-tight joints with the ends of the étalon, the internal space may be evacuated, if desired, through a suitable nipple fitted in the étalon wall.

(*b*) *Optical Multiplication.*—In the next stage the first étalon is optically compared with another étalon of similar construction whose length is a simple multiple of that of the first, and in the following stage the second étalon is similarly compared with a third, again of similar construction, whose length bears a simple multiple relation to that of the second, the process being repeated until an étalon is arrived at whose length is approximately equal to the fundamental unit of length. These optical comparisons are performed with the aid of BREWSTER's fringes, or the fringes of superposition, which are observed by means of a telescope, focussed for parallel light, in a beam of white light transmitted through two étalons in series.

BENOÎT, FABRY and PEROT carried out their optical multiplication on a binary scale, so that a series of five étalons was required to arrive at a length of one metre, starting from 1/16-metre. In addition, two optical wedges or compensators were used by them to determine the departures of their series of étalons from an exact binary scale.

FABRY and BUISSON* have since pointed out that the number of stages of multiplication may be reduced and that the use of the optical wedge or compensator is not essential. They showed that compensation for errors in the lengths of two étalons may be effected by inclining one étalon with respect to the other. The suggested

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* (*a*) FABRY and BUISSON, 'J. Physique,' vol. 9, p. 189 (1919). (*b*) FABRY, "Les Applications des Interférences Lumineuses," p. 82 (1923 edition).

modification enabled FABRY and BUISSON to compare directly two étalons of lengths in the ratio of 10 to 1.

Full discussion of the practical application of BREWSTER's fringes to optical multiplication of length is unnecessary here, for the subject has been adequately discussed in original papers by the authors mentioned. It is therefore only necessary to refer here to one or two important considerations. Suppose that, in fig. 1, l_1 and l_2 represent respectively the optical lengths of two étalons E_1 and E_2, l_2 being slightly greater than $4l_1$. Suppose also that a convergent beam of white light, incident from the left of the figure, passes first through E_2 and then E_1 and enters objective O of a telescope focussed for parallel light. The étalon E_1 is adjusted previously, by auto-collimation, so that its partially reflecting surfaces are accurately normal to the optical axis of the telescope. Then, by inclining étalon E_2 with respect to this axis, the BREWSTER's fringes appear in the telescope field, taking the form of straight bands parallel to the lines of intersection of the reflecting surfaces of the two étalons. As the inclination of E_2 is increased, the fringes move across the field keeping parallel to themselves, while their separation decreases, and a condition is presently reached at which the central white fringe coincides with a cross-wire at the centre of the telescope field. If α is the angle between the axes of the two étalons at this stage, then :—

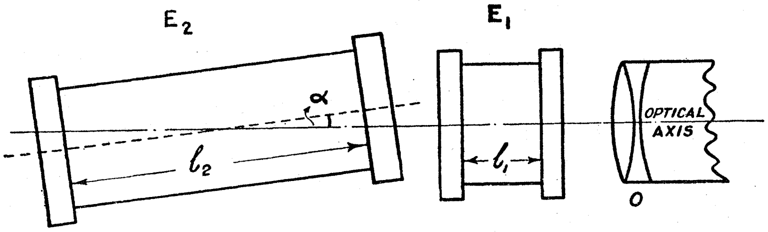

FIG. 1.—Scheme of optical multiplication.

$$4l_1 = l_2 \cos \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It can also be shown* that the angular separation of two adjacent fringes in the condition when the central white fringe intersects the optical axis of the telescope is given by θ, where :—

$$\theta = \frac{\lambda}{2l_2\alpha}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The value of the fringe separation θ may be expressed in terms of λ, l_2 and l_1 by substituting for α in equation (2) its value derived from (1), whence, assuming that α is small :—

$$\theta = \lambda \left[8l_2 (l_2 - 4l_1)\right]^{-\frac{1}{2}}$$

or, if l_2 is slightly greater than nl_1, then :—

$$\theta = \lambda \left[8l_2 (l_2 - nl_1)\right]^{-\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

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* See FABRY, *loc. cit.*, p. 83.

For a given telescope magnification, there will be an optimum value of θ corresponding to the condition that the fringes are not too broad and diffuse for accurate setting of the central fringe to a cross-wire, and not so closely spaced that it is difficult to distinguish the central fringe. In order, therefore, to make the best use of the fringes it is of great importance that the lengths of E_2 and E_1 may be subject to control, and the straining wires fitted to the étalons provide a sensitive control of their relative lengths within suitable limits.

Another important consideration in the practical application of the fringes is the determination of the maximum degree of multiplication which may be performed in one operation without materially increasing the experimental error. Referring to fig. 1, it is obvious that, in the simplest case, equivalence of path difference is only attained (when $l_2 = 4l_1$) by those rays which suffer two internal reflections in E_2 and pass directly through E_1 and other rays which pass directly through E_2 and suffer eight internal reflections in E_1. In practice the intensities of the interfering rays are usually unequal, and the resulting fringes are consequently of lower visibility than if the intensities were equal. As the degree of multiplication performed in one stage increases so the visibility of the resultant fringes decreases. Indeed, it has been shown* that if f is the common reflection coefficient of the four semi-silvered surfaces and the ratio of lengths is $m : n$, then the aspect of the resultant fringes is exactly the same as would be produced by a simple wedge interferometer with two semi-silvered surfaces each having a reflection coefficient of $f^{(m+n)}$.

In a recent publication,† FABRY expressed the view that it is possible to proceed from an étalon of length directly measurable in terms of the red radiation of cadmium up to an étalon one metre long in two stages of optical multiplication. Evidence regarding this problem was obtained with experimental étalons, before the present apparatus was designed, and it was decided that two stages were necessary and sufficient. At the same time the opinion was formed that, provided the relative lengths of the étalons could be precisely controlled, the aspect of the fringes in the two stages of comparison could be adjusted so that settings of the central white fringe to a cross-wire could be repeated to an accuracy of about 0·01 to 0·02 of the fringe separation.

Thus the second stage of the present procedure consists of a comparison of the 1/12-metre étalon, which is directly measurable in terms of wave-lengths, with an étalon of length 1/3-metre, utilising BREWSTER's fringes as an indicator. Compensation of small errors in the simple numerical ratio between the lengths of the two étalons is obtained by inclining the 1/3-metre étalon through a measured angle from the optical axis defined by the first étalon and the telescope. If the longer étalon (1/9-metre) is used for the basic measurement, the optical comparison with the 1/3-metre étalon is performed in the same way, but the ratio of lengths is reduced from 4 : 1 to 3 : 1.

In the third stage the 1/3-metre étalon is compared in exactly the same manner

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* See BENOÎT, FABRY and PEROT, *loc. cit.*, p. 32.

† See FABRY, *loc. cit.* (*b*), p. 88.

with the 1-metre étalon, compensation as before being effected by suitable inclination of the 1/3-metre étalon.

(*c*) *Optical Measurement of an End-Gauge.*—The subsequent operation in the present work shows a distinct change from the procedure adopted by the earlier observers. Whereas Benoît, Fabry and Perot compared their optically determined metre étalon with a line-standard by the normal methods of microscope observations in a line-standard comparator, with the aid of fine lines scribed on the upper edges of the terminal glass plates of the metre étalon, the present method derives the length of an end-standard directly from the length of the metre étalon by optical interference methods. Thus, one of the most difficult steps in the earlier work is eliminated, *i.e.*, that relating to the correlation of the positions of the reflecting surfaces of the metre étalon with those of the graduation lines on the upper edges of the terminal plates.

The end-gauge used for this purpose is constructed in the form of an X-section steel bar, with flat polished ends, which is supported, during its optical measurement, inside the bore of the longest étalon, so that the four channels formed between the arms of the X and the étalon wall are available for the optical comparison of the longest étalon with the intermediate étalon. The arrangement for supporting the gauge along the bore of the étalon is carefully designed and made, so that the ends of the gauge are optically parallel to the terminal surfaces of the étalon. Since it was desired to compare both yard and metre lengths with the wave-length of light in the same apparatus the étalon is made slightly longer than a metre. In order to determine the optical length of the end-gauge it is necessary to measure in terms of light waves, firstly the optical length of the étalon along each of the four channels, and secondly the gaps left at each end between the terminal surfaces of the gauge and of the étalon, and to subtract the results. Since the measurements of the end gaps are made in light reflected from the central portions of the end surfaces of the étalon and gauge, a separate experiment is conducted with the gauge removed in order to determine the correction necessary to relate the axial length of the étalon to its mean length as measured along the four channels. The lengths of the gaps for both yard and metre gauges fall easily within the range directly measurable in terms of light waves by the methods usually applied to Fabry-Perot étalons.

The value obtained for the length of the X-gauge is the optical length of the gauge, and this differs from the mechanical or practical length by reason of the loss of phase suffered by waves reflected at the polished steel surfaces. The correction for this effect has been determined by separate experiment on similarly treated steel surfaces, and an account of this work has been published elsewhere.* There is a further small correction for a similar loss of phase at the silvered plates on the longest étalon, which arises because they are used in a slightly different manner in optical multiplication than when they are used in the determination of the gaps. Further reference to this is made later.

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* Rolt and Barrell 'Proc. Roy. Soc.,' A, vol. 122, p. 122 (1929).

In fig. 2, which is a general diagram in plan view of the optical arrangement of the apparatus, it will be seen that the three étalons taking part in the standardisation of the X-gauge are set up in line with the main telescope. This telescope is used for observing all interference phenomena except those concerned in the measurement of the gaps at the ends of the gauge. It is necessary that the shortest étalon be capable of a sideways displacement in order to enable the main telescope to be used for the optical comparison of the two longer étalons. Means are provided also for displacing the longest étalon so that the four channels may be individually compared with the intermediate étalon. Compensation in the optical comparisons is performed entirely with the latter étalon, and for this purpose it is mounted on a support capable of giving it small inclinations, in a vertical plane, through measured angles from the optical axis of the line of étalons and the telescope.

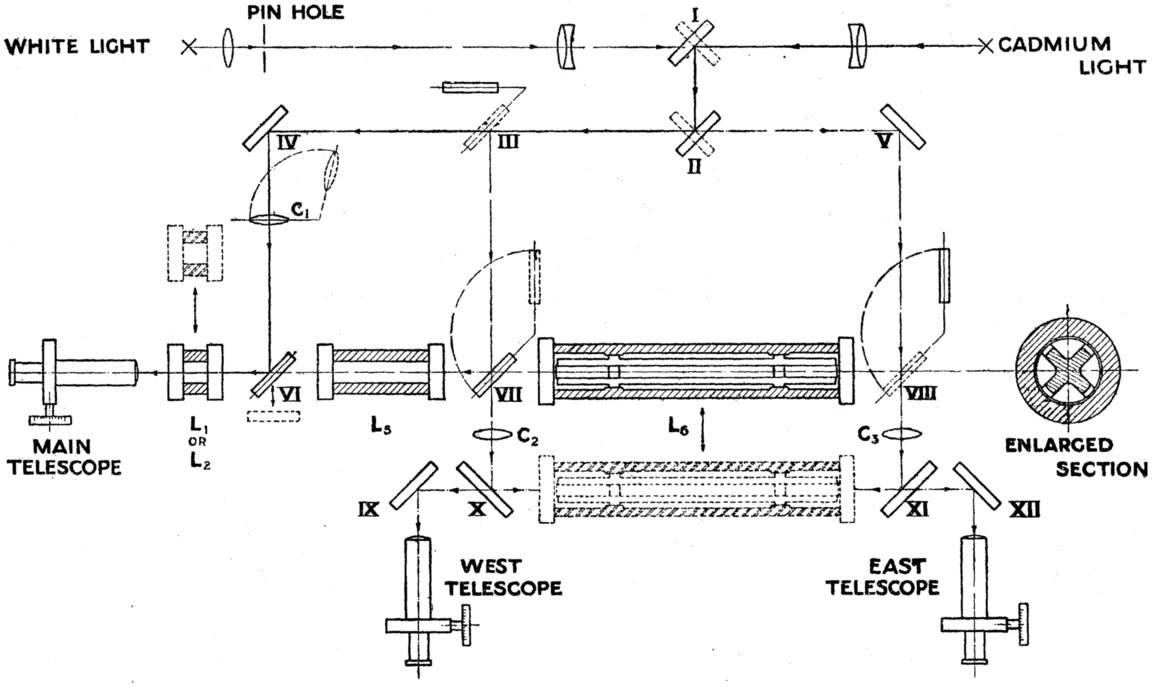

Fig. 2.—Optical arrangement of apparatus.

For the measurement of the gaps at the ends of the gauge, the complete assembly of the longest étalon with the contained X-gauge is displaced laterally to a position between the two systems of mirrors and telescopes shown in the lower part of the diagram.

Since all the étalons are of the same type, capable either of evacuation or of being filled with air of known composition, the measurement of the gauge can be performed either in vacuum or under definitely controlled atmospheric conditions.

(*d*) *Temperature.*—It may be thought that a considerable proportion of the advantage accruing from the use of invar étalons has been lost unnecessarily by constructing the X-gauge of steel, which has a thermal coefficient of expansion about ten times greater than that of invar. But steel bars with hardened and polished terminal faces have been found by experience to be the most suitable form of end-gauge for practical purposes. Therefore, as all gauges with which the X-gauge is to be subsequently compared are likely to be made of steel, it is undoubtedly best to construct it also of steel, for means of accurate control and measurement of its temperature may be more conveniently applied during its wave-length standardisation than during subsequent comparisons. In other words, if the temperature at which the steel X-gauge is determined in terms of light waves is accurately known, then, in its subsequent comparisons with other steel gauges, owing to the similarity of the thermal coefficients of expansion involved, the question of accurate temperature measurement does not enter to the same degree as it would do if the X-gauge were made of invar.

Since an accuracy of about 1 part in 40×10^6 in the measurement of lengths of one yard and one metre is the ultimate aim of the present investigation, the temperature of the steel X-gauge must during the optical determinations be measured to an accuracy of at least $\pm 0 \cdot 002_5$°C., since the thermal coefficient of expansion of steel is rather more than 10×10^{-6} per 1° C.

There are two essential conditions to be satisfied before the temperature of the gauge can be truly defined with such high precision. First, the region occupied by the gauge must be maintained continuously at constant temperature to a similar order of accuracy in order to eliminate the effects of lag on the readings of a thermometer, which must be used to indicate the temperature of the gauge. Secondly, the temperature in this region must be spatially uniform to a similar order of accuracy to ensure that the thermometer reads the true mean temperature of the gauge. The most practicable method of satisfying these conditions in such an extensive system of apparatus is to cover the apparatus with a heat-insulating enclosure, and to treat the whole enclosure as an air thermostat.

Referring again to fig. 2, the system of five mirrors together with the sources of light and their associated optical apparatus, shown at the top of the diagram, are mounted on a slate bench fixed to one wall of the room in which the rest of the apparatus is built. To achieve stability and thermal insulation, the latter part, comprising étalons, telescopes and other auxiliary mirrors and lenses, is mounted on the slate top of an isolated concrete foundation extending 6 ft. into the ground, and is covered by a double-walled, lagged wooden case resting on the floor of the room. The case is built to be as air-tight as possible, and the temperature of the air within it is maintained, by electrical heaters under the control of a sensitive toluene-mercury regulator, at a value somewhat higher than that of the room. The room itself is an inner room and is also thermostatically controlled by similar systems at a temperature slightly above its surroundings. Uniformity of temperature within the case is obtained by means of a fan which vigorously circulates the air about the enclosed apparatus, and the room is stirred by another fan. The lid and part of one side of the case are removable in sections to allow easy access to the apparatus when its temperature is not being controlled. Double glazed windows are provided in the walls to admit the passage of the necessary beams of light, together with hand-holes fitted with air-tight wooden plugs to enable certain adjustments to be made when the apparatus is under controlled conditions. Various levers and cable controls, required in the operation of the apparatus during observations, are carried through the walls of the case for external manipulation. Fig. 21, Plate 7, is a photograph of the whole apparatus, and gives a good general impression of its lay-out.

Measurement of temperature is carried out by means of platinum resistance thermometers. The most important of these, called Θ, is actually incorporated in the longest étalon, for this contains the steel gauge, and it is the temperature of this gauge at the time of its comparison with the wave-length which needs to be most accurately measured. Two auxiliary platinum resistance thermometers, known as T_1 and T_2, are suspended from a rail inside the lid of the case; they can be moved along the rail to observe the degree of uniformity of temperature attained throughout the case before observations commence, and during the observations one of them is left in close proximity to the shortest étalon in order to measure the temperature at which the basic measurement of length is made. This last measurement of temperature is only of importance when the étalon is filled with air, and is not strictly necessary when it is evacuated, provided that the temperature remains constant within, say, 0·01° C. during the observations.

(*e*) *Comparison of End-Gauge with Prototype.*—Having now outlined the method whereby an end-gauge equal in length to one of the units of length is compared with the wave-length of light, and mentioned some of the major precautions necessary to secure the desired accuracy of comparison, it remains to describe the final links in the complete comparison of the wave-length with the existing standards of length. Since the existing standards are line-standards, use is made of a special composite gauge to determine the relationship between the X-gauge and the Prototypes. The composite gauge has been described elsewhere,* and has been in use for many years at the National Physical Laboratory. It consists of an auxiliary end-bar of circular section and of length slightly (say half-an-inch) shorter than that of the line-standard, together with two parallel-faced blocks each half an inch in thickness, with sides equal respectively to the radius and diameter of the bar. At the middle of the $\frac{1}{2}$-inch face of each block a fine line is ruled parallel to the longer edge. The two blocks are then wrung on to the two ends of the bar, in such a manner that the graduated faces are parallel to each other and in the neutral plane of the bar.

A series of comparisons is made under microscopes of the distances between the lines on the composite gauge and between those on the line-standard, reversing the blocks between the measurements so that their opposite faces are in turn in contact with the ends of the bar. The average of all these measurements gives the difference

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* "Annual Report of the N.P.L.," p. 90 (1919).

between the lengths of the line-standard and of the auxiliary end-bar plus half the sum of the blocks. The auxiliary bar, with one block at a time wrung on centrally at its end, is then compared against the end-standard, and the average of these measurements gives the difference between the end-standard and the auxiliary bar plus half the sum of the blocks. Subtracting the two results, the lengths of the auxiliary bar and blocks are eliminated and the difference between the end-standard and the line-standard is ascertained.

The process of comparing the length of the X-gauge with that of the special composite gauge used as an end-gauge is performed optically by a simple extension of the method already described for deriving the length of the X-gauge from the length of the longest étalon. Suppose that the X-gauge is introduced between a pair of semi-silvered mirrors independently adjustable into parallelism with the terminal surfaces of the gauge ; then another end-gauge with faces of similar finish, and not too different from it in length, may be compared with it by interchanging the two gauges under suitable conditions and by determining optically, in sequence, the parallel-faced gaps between the ends of the gauges and the two mirrors.

An optical comparator has been designed capable of performing this comparison in a satisfactory manner, and is included in the apparatus situated in the thermally controlled enclosure. Fig. 22, Plate 7, is a photograph of the comparator, with one of the end mirror holders removed, arranged for the comparison of the X-gauge with the composite gauge.

In brief, then, the method of comparing a wave-length of light with one of the present fundamental standards of length consists of a minimum of seven operations :—

(*a*) Determination of the number of waves of a monochromatic radiation contained in a Fabry-Perot étalon approaching 10 cm. in length ;

(*b*) Optical comparison of the first étalon with a second of intermediate length by the use of BREWSTER's fringes in white light ;

(*c*) Optical comparison of the second étalon with a third, of length somewhat greater than the fundamental unit of length, by the same method as in (*b*) ;

(*d*) Direct determination of the difference between the lengths of the third étalon and an end-gauge of length nominally equal to one of the fundamental units, in terms of the same monochromatic radiation as used in (*a*) ;

(*e*) Direct optical comparison of the end-gauge mentioned in (*d*) with a composite gauge of special construction, in terms of the same monochromatic radiation ;

(*f*) Comparison of the composite gauge, in a line-standard comparator, with a reference line-standard which has been compared with the Prototype ;

(*g*) Comparison of the reference standard with the Prototype in a line-standard comparator.

With regard to the last two operations, the introduction of the reference line-standard, or Prototype copy, is rendered necessary by the fact that the use of the

Prototypes is legally restricted to comparisons with their copies at certain prescribed times, and that it is the copies which serve as the bases of comparison during intermediate periods.

For the provisional measurements of the Metre, the results of which are quoted later in this paper, two further stages were involved between (*f*) and (*g*). The intermediate line-standard employed for comparison with the composite end-bar was the nickel bar which constitutes the reference standard of the Laboratory. This bar has been compared with the working standards of the Bureau International both directly and through the agency of the British National Copy of the Metre. The working standards of the Bureau are in their turn compared with the "Témoins" of the International Metre, and these finally are compared with the Prototype Metre itself.

Fig. 23, Plate 8, is a photograph of the collection of four étalons (the two shortest being alternatives), the X-gauge of one metre length, the special composite gauge with the second block lying on its right, and the reference line-standard in the foreground, which illustrates in a striking manner the number of links in the complete comparison of the wave-length of light with an existing standard of length.

3. *Apparatus.*

(*a*) *Optical Arrangement.*—In the diagram of the optical arrangement of the apparatus given in fig. 2, L_6 is the longest étalon which carries the X-gauge along its bore, L_5 is the intermediate étalon, of length approximately equal to $L_6/3$, and L_1 is the étalon, of length approximately equal to $L_6/12$, which is used for the basic measurement in terms of light waves. Another étalon L_2, of length approximately equal to $L_6/9$, may be used as an alternative to L_1 for the basic measurement. Two other étalons were contemplated in the original scheme, namely, L_4, approximately equal to $L_6/4$, to be used as an alternative to L_5 in the optical multiplication from L_1 to L_6, and L_3, approximately equal to $L_6/6$, to be directly measured in terms of light waves, if suitable monochromatic radiations could be found, and then optically compared in one stage with L_6 using the fringes of superposition as in the other optical comparisons. Neither L_3 nor L_4 has yet been constructed.

The system of five mirrors, at the top of fig. 2, which are mounted upon an independent shelf, away from the pillar which supports the main apparatus, enables either white or monochromatic light to be introduced to the line of étalons at three different points at will. Three other mirrors, one situated to the right of each étalon, further direct the light along the line of the étalons. Referring to the diagram, mirror I has two positions, controlled by stops, which allow it to collect light from either the white or the monochromatic source and direct it on to a similar mirror II; the latter directs the light on to mirrors III, IV or V. Mirror III has "in" and "out" positions, mirrors IV and V are fixed, while mirrors VI, VII and VIII have "in" and "out" positions.

In the normal operation of the apparatus the étalon L_1 (or L_2) is first illuminated with monochromatic light, which follows the course indicated by the continuous line in fig. 2. The interference phenomena are observed in the main telescope. When the two étalons L_1 (or L_2) and L_5 are optically compared in white light, mirror VI is turned to its "out" position, mirrors I, III and VII are turned into appropriate positions to direct white light through the two étalons, and the fringes of superposition are observed in the main telescope. Optical comparison of étalons L_5 and L_6 is effected by rotating mirror VII to its "out" position and mirrors II and VIII to appropriate positions, so that white light is directed through these étalons, étalon L_1 being displaced transversely from its path.

In order to determine the gaps existing between the X-gauge and the terminal plates on étalon L_6, the complete assembly of étalon and gauge is displaced to a position between the two mirror systems IX, X and XI, XII. Mirrors VII and VIII are rotated to their "out" positions and suitable changes are made in other mirrors so that either mirror X or XI is illuminated with monochromatic light. Mirrors X and XI are plane parallel glass plates, each semi-silvered on one surface, and set at 45° to the étalon axis. Confining attention to a parallel-faced gap at one end of the étalon assembly, the convergent light partially reflected from mirror X falls normally on the gap. Here, the action of the two reflecting surfaces gives rise to interference rings which are observed through mirror X by the west telescope, mirror IX merely directing the interfering beams into the telescope. The same action takes place at the other gap, where the interference rings are observed through the east telescope.

Distances between sources and observing points are unavoidably great, and undue loss of light is prevented by using parallel beams of light of sufficient aperture to fill the mirrors. Thus, light from the monochromatic source is rendered parallel by an achromatic lens of 45 cm. focal length. Before reaching the étalon L_1 (or L_2) or the parallel-faced gaps in étalon L_6, the monochromatic beams are given the necessary convergence by auxiliary lenses C_1, C_2 and C_3 of the spectacle lens type. C_1 has a focal length of about 130 cm. and may be turned out of the light path if required, while C_2 and C_3 are each of 30 cm. focal length. The source of white light is a 500 c.p. Pointolite lamp having an illuminating electrode in the shape of a diamond with 6 mm. side. During preliminary adjustments of the étalons to parallelism, for which accurately parallel beams of light are required, an image of this source is formed at a pinhole. The illuminated pinhole, together with an achromatic lens of 150 cm. focal length, provides the necessary parallel beam. During optical comparisons of the étalons the pinhole is removed and the fringes of superposition are viewed on an image of the extended source seen through the main telescope.

The main telescope has an objective of 43 cm. focal length. It is fitted with a micrometer eyepiece giving a magnification of about ten times. An auxiliary Gauss eyepiece, interchangeable with the ordinary eyepiece, is used in association with the telescope for auto-collimation purposes. The micrometer eyepiece has a pair of fixed cross-wires, one vertical and the other horizontal, whose intersection defines the optical centre of the telescope field, and also a movable doublet under the control of the micrometer screw and drum. Fibres of glass silk have been found to make very suitable cross-wires for the type of optical setting which has to be made. The micrometer screw has a pitch of 0·25 mm. and the drum is divided into 100 parts. Similar micrometer eyepieces are fitted to the east and west telescopes, each of which have interchangeable objectives of 25 cm. and 10 cm. focal lengths. As the gaps at the ends of the metre X-gauge are much smaller than those at the ends of the yard X-gauge, less magnification is required for observing the interference rings at the ends of the metre gauge, and therefore the objectives of smaller focal length are used in the measurement of the metre gauge.

(*b*) *The Étalons and X-Gauge.*—Preliminary trials showed that the ends of invar tubes could not be brought to satisfactory optical flatness and parallelism by the usual methods, owing to the comparative softness of invar. Part of the finishing process normally applied to hardened steel end-gauges consists of lapping the terminal surfaces on a flat cast-iron plate or lap impregnated with very fine abrasive material. Hardened steel surfaces do not pick up the abrasive material, whereas invar surfaces do. Consequently, when attempts were made to bring lapped invar surfaces into wringing contact with glass or quartz plates, the contact was rendered uncertain, or even impossible to attain, because of the presence of abrasive particles.

Attention was then turned to the electro-deposition of chromium upon invar surfaces. By using special methods chromium was deposited directly upon invar in a practically uniform layer, of sufficient thickness and hardness to permit the attainment, by the application of the usual methods of lapping and polishing, of surfaces of optical flatness and suitable for wringing. Unfortunately, chromium directly deposited on invar usually tended to crack and flake away in course of time, and the method had to be modified. It was considered that the instability was due to the large difference in hardness between invar and chromium, and that the difficulty might be overcome by depositing on the invar a series of metals of gradually increasing hardness, such as copper, nickel and chromium. Experiments on these lines were successful in producing a final chromium deposit on which a precise optical surface could be worked. The surface was quite permanent and eminently suitable for wringing. Only in one case was trouble experienced owing to the development of a small blister on one of the plated surfaces subsequently to lapping, where it appeared that the nickel deposit had parted locally from the copper. The final surfaces on the invar étalons are somewhat delicate compared with hardened steel surfaces, owing to the softness of the invar base, but with reasonable care they withstand numerous repetitions of the wringing process without suffering damage. A description of the electro-deposition process and the gauging methods applied to control sizes during the process is given in Appendix I.

All the étalons except L_6 were constructed of similar invar tubing having an external diameter of 35 mm. and a bore of 19 mm. diameter. A diagram of the complete étalon L_2 is shown in fig. 3. The étalon normally rests in a V-groove, in which it is supported on the edges of two removable invar flanges which abut against smaller flanges machined on the outer wall of the tube. The portion between the flanges may be elastically compressed, and consequently shortened, by drawing the flanges together with the aid of four invar straining wires symmetrically disposed around the étalon. It can be understood, by reference to the figure, how the length and parallelism of the étalon are adjusted by tightening up the square nuts against the right-hand flange. Small pins fixed to the straining wires and engaging in slots in the right-hand flange prevent the wires from rotating during the straining process. There are also small pins fixed in the outer wall of the étalon for the purpose of locating the removable flanges in their correct relative positions. Since the total cross-sectional area of the four straining wires is much less than that of the tube the method of adjustment is extremely sensitive; and it is capable of producing a change in length of any étalon of about 1 part in 25,000 without exceeding the elastic limit of the straining wires. Connection to a vacuum pipe line or a controlled atmosphere is made through the nipple shown in the diagram.

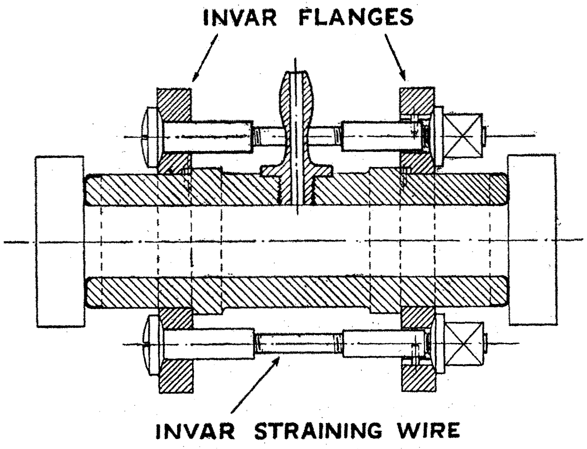

FIG. 3.—Étalon L_2 (drawn to scale).

A diagram of the longest étalon L_6 is shown in fig. 4, which is not drawn to scale. This étalon has an external diameter of 57 mm. and a bore of 43 mm. The portion between the flanges is fitted with a platinum resistance thermometer, wound on mica strips fixed into slots on the external surface, and protected by a brass tube. A fuller description of this feature of the étalon is given in Appendix II, which deals with the measurement of temperature. Control of length and parallelism is made by means of straining wires as already described and a vacuum connection is provided near one end of the étalon.

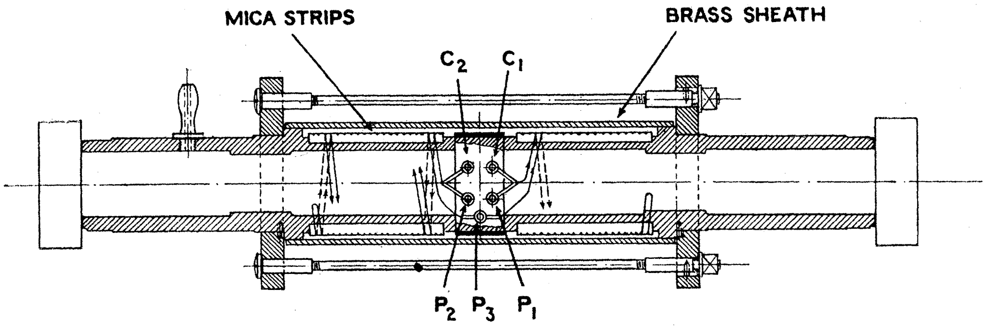

FIG. 4.—Étalon L_6 (not drawn to scale).

Table I gives the finished lengths, in the unstrained condition, of the four étalons which have so far been constructed.

TABLE I.—*Finished Lengths of the Four Étalons.*

L_6	1006·5289 mm.	L_2	111·8350 mm.
L_5	335·5037 mm.	L_1	83·8762 mm.

The errors of flatness and parallelism of the end surfaces of the étalons when finished and in the unstrained condition did not generally exceed $0{\cdot}15\ \mu$, and the adjustment to exact parallelism in each étalon and to correct relative lengths for the scheme of optical multiplication was easily within the control of the straining wire system fitted to the étalons.

It will be observed in fig. 4 that there are two raised bands in the bore of the longest étalon situated near the removable flanges. These internal bands were ground and lapped as accurately as possible to be truly cylindrical, co-axial, and equal in diameter, and during the final finishing process the end surfaces of the étalon were produced as accurately as possible normal to the axis defined by the internal bands. The X-section end-gauge to be optically standardised also possesses two external bands which were ground as accurately as possible to be truly cylindrical, co-axial and equal in diameter. Their separation on the gauge was calculated by means of the usual Airy formula $L/\sqrt{3}$, where L is the length of the gauge. Using the same procedure as for the étalon, the terminal faces of the gauge were optically finished to be accurately normal to the axis defined by the Airy bands and were, therefore, parallel to one another. This operation was actually performed with the axis of the gauge vertical. It is the important property of the position of the Airy bands that when a gauge is produced thus with its terminal surfaces parallel in an unstressed condition they remain parallel when the gauge is supported horizontally on the bands and slightly flexed by the influence of its own weight. Thus, when the gauge and étalon are assembled together, with the gauge supported on its bands by the internal bands of the étalon, the terminal faces of the gauge and étalon are mutually parallel. It is essential for easy assembly that the gauge bands be slightly (*e.g.*, 0·025 mm.) less in diameter than the étalon bands. Accommodation for either the yard or metre end-gauge was provided in the same étalon by forming sufficiently long internal bands in the étalon.

The diagram, fig. 2, shows the general arrangement of the X-gauge and the étalon, the form of the channels left for comparison of L_6 with L_5 in white light being given in the enlarged section on the right.

At present only the X-gauge of a metre length has been completed. The corresponding yard gauge is in course of construction. Concerning the accuracy of construction of the metre gauge, it is of interest to mention that the central areas 1 cm. in diameter at each terminal surface, the mean optical separation of which actually defines the length of the gauge in the wave-length comparisons, are flat, parallel to one another, and square to the axis defined by the Airy bands, all to an accuracy within $0{\cdot}05\,\mu$. The mechanical length of the gauge is about $1\,\mu$ in excess of 1 metre, but since it is not important for the purpose in view that the length should be an exact reproduction of the metre no attempt has been made to reduce it further.

The whole of the operations of final finishing of the terminal surfaces of the étalons and X-gauge was performed by The Pitter Gauge & Precision Tool Co., of Woolwich.

The étalons of smaller bore are used in conjunction with optically flat glass plates about 13 mm. thick—a value found by direct experiment to reduce to negligible proportions the distortion produced when the étalons are evacuated. The plates on the longest étalon are 25 mm. thick and are made of natural quartz; this material, by reason of its hardness, is better able to withstand the more frequent wringing operations to which the longest étalon is ordinarily subjected. Each glass or quartz plate was finished with a small angle of about 2 minutes between its surfaces. The pairs of plates have optically similar angles, and each pair is wrung to an étalon so that the angles mutually oppose one another. This is the usual practice in Fabry-Perot étalons for avoiding confusion by extraneous interference phenomena which would arise from the presence of four mutually parallel surfaces.

(*c*) *Étalon Supports.*—The main body of the apparatus is mounted on the top of a long concrete pillar on an independent foundation isolated from the rest of the building. At one end a heavy cast-iron base, with three levelling screws geometrically located on the slate top of the pillar, carries the main telescope, the first two étalons, and the mirror between these étalons. Figs. 5 (*a*) and 5 (*b*) show respectively elevation and plan views of this assembly, omitting, for the sake of clarity, the mirror between the étalons, and other irrelevant details.

The telescope is carried in two rings on a separate casting which is bolted down to the main base. Fine adjustments to the alignment of the telescope are made by three set screws in each ring.

The first étalon lies in a V-groove on another separate unit. A more detailed view of this unit, as seen in part section from the right of fig. 5, is given in fig. 6. In order to prevent the interception of light by the first étalon during optical comparisons of the

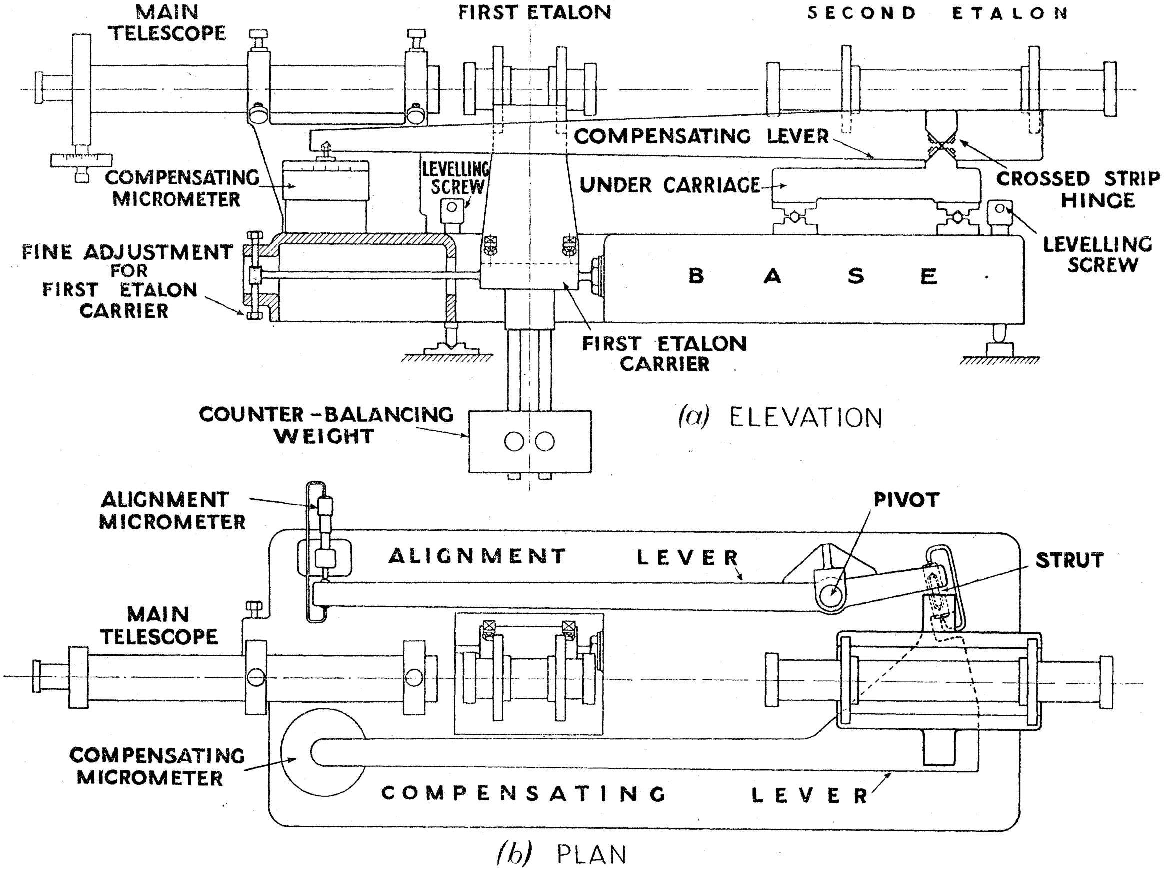

Fig. 5.—Support for first two étalons and main telescope.

other étalons, the carrier for the first étalon is designed to hang on a horizontal cylinder formed on a steel rod. There is a clearance hole in the main base and the concrete pillar which allows the carrier to rotate about the cylinder between adjustable stops. The position of the counterbalancing weight is arranged so that the system is slightly top-heavy, and the carrier can then make definite location against the stops in either position. Both coarse and fine adjustments to the axial alignment of the étalon are provided ; the coarse adjustment is effected by means of four levelling screws (of which only two are seen in fig. 6) bearing on the cylinder ; fine adjustment is obtained by bending the support of the cylinder at its fixed end. Referring to fig. 5 (*a*), it will be seen that the steel rod is actually bent at the portion near its fixed end where its diameter is much reduced, and the amount of bending is controlled by four set screws acting on a boss at the extreme left of an extension from the cylinder. The sensitivity of the adjustment is greatly increased by the elasticity of this extension, which is also of reduced section. Access to the four set screws is provided through a hand-hole in the wall of the case. The movement of the étalon carrier is controlled from outside the case by means of a lever, not shown in the diagrams, which operates so that there is no tendency to twist the carrier about a vertical axis. Once adjusted, the étalon may be displaced from its normal position and returned again without causing any disturbance of its alignment.

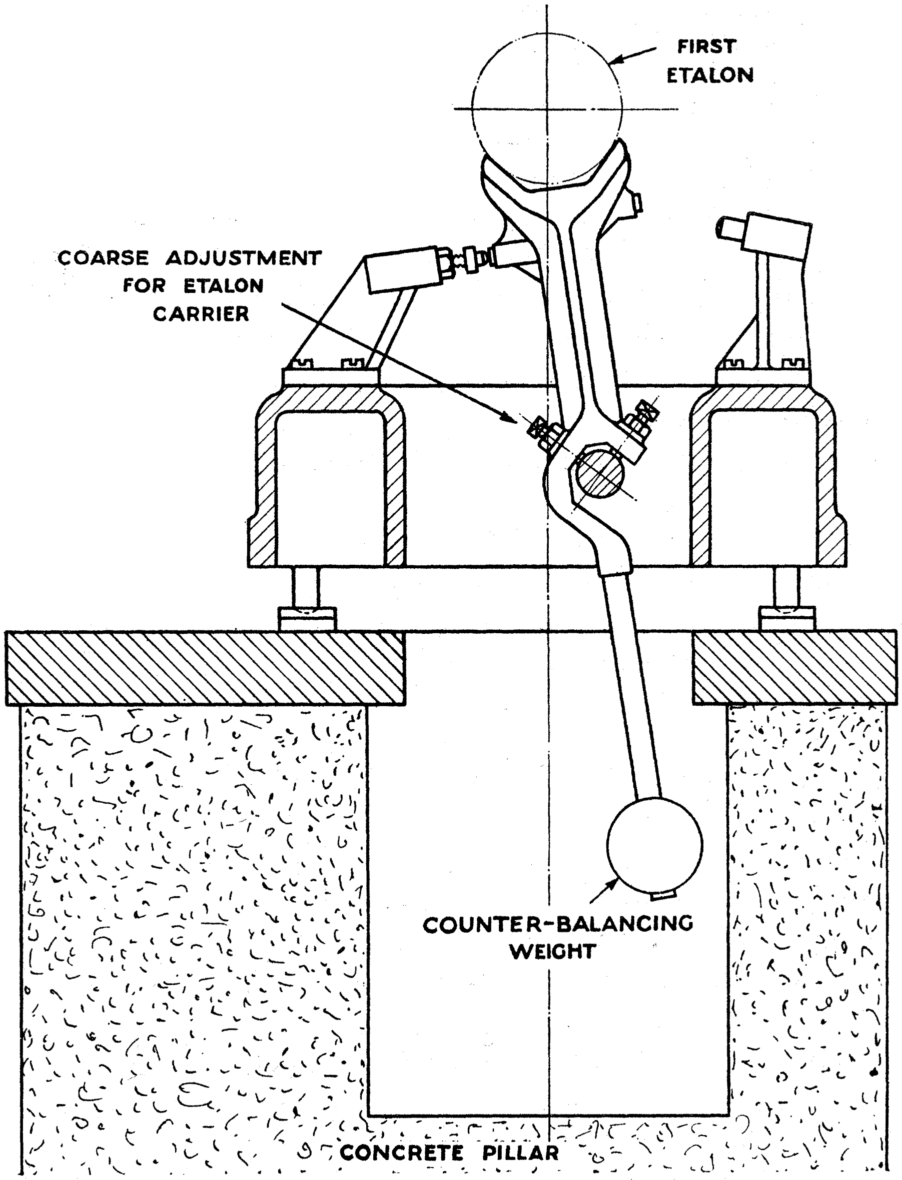

Fig. 6.—Displacement system for first étalon.

In order to enable the length of the second, or intermediate, étalon to be compared with that of the first or third étalons, it is supported in a V-groove formed on a casting which can be given small rotations about a horizontal axis at right angles to the axis of the groove. The complete support for the second étalon is made in two parts. The upper casting, which is of light alloy, carries the V-groove and also a long arm, called the compensating lever, which is connected by a " crossed-strip " hinge* to an under-

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* Rolt, " Gauges and Fine Measurements," vol. 2, p. 281 (1929 edition).

carriage. The hinge allows rotation of the V-groove to take place about an axis defined by the line of intersection of the median planes of the crossed steel strips, and the amount of rotation is controlled by the compensating lever, the end of which engages through a ball-ended strut with an accurate micrometer screw. The pivot formed by the hinge is frictionless and entirely free from backlash.

The under-carriage has on its lower surface three inverted V-grooves arranged immediately above and parallel to three similar grooves on the main base. Each pair of grooves contains a steel ball. The grooves are arranged with the directions of their lengths at right angles to the lines joining their centres to the axis of the micrometer screw. Consequently, the whole carriage can be brought into horizontal alignment by means of small rotations about the vertical axis of the micrometer screw imparted by the alignment lever mechanism shown in fig. 5 (*b*). This lever is mounted on an ordinary pivot, and movements of its long end, controlled by a small micrometer, are transmitted to the under-carriage through a ball-ended strut at the short end.

The two lever systems thus allow the second étalon to be adjusted about two axes at right angles. For optical comparisons of this étalon with the first or third étalons it is first adjusted for horizontal alignment, and compensation of the optical paths is then obtained by inclining it suitably in the vertical plane with the aid of the compensating lever mechanism. The micrometer which moves the latter has both fast and slow motion controls operated by knobs from outside the lagged wooden enclosure at a position near the main telescope. Since the effective length of the lever from hinge to thrust is very closely 25 inches, and the pitch of the micrometer screw is $0 \cdot 05$ inch, each revolution of the screw corresponds nominally to a change of $0 \cdot 002$ radian in the inclination. The screw is fitted with an enlarged drum divided into 200 divisions, and one division therefore corresponds to a nominal change of 1×10^{-5} radian. A vernier permits of readings to 1×10^{-6} radian. The drum and vernier are suitably illuminated through a window in the lid of the enclosure, and a reading lens fitted in its end wall enables them to be observed.

As the accurate measurement of the angle of compensation is of great importance in the scheme of optical multiplication, the support for the second étalon, after careful adjustment, was calibrated in the following manner.

The value of one division on the compensating micrometer drum was determined by measuring the angles of inclination of the compensating lever corresponding to definite displacements of the micrometer over four different parts of its range. The angles were measured by means of a sensitive spirit level, and the value of one division was found to be equal to $0 \cdot 000{,}010{,}00_0$ radian, the results of the individual displacements agreeing among themselves to within one part in 5,000. This calibration was independently checked by purely mechanical measurements of the effective length of the lever and the pitch of the screw. The compensating micrometer screw was examined for progressive error at alternate revolutions over its range. At no position did the progressive error exceed $0 \cdot 1$ division on the drum, which is equivalent to $0 \cdot 000{,}001$ radian. The periodic error in the screw was examined at five positions over one revolution and nowhere did the error exceed ± 0·1 division.

The supporting mechanism for the longest étalon must be capable of very accurate parallel displacements in order that the four channels, existing between the arms of the X-gauge and the wall of the étalon, may be individually compared with the intermediate étalon without disturbance of its original alignment. It must also be capable of a larger parallel displacement to a position where the parallel-faced gaps at the ends of the étalon assembly are measured. A diagram of the support for this étalon appears in fig. 7. It consists of two main castings of light alloy. The upper casting, called the cradle, is of rectangular formation, with V-grooves running the full length of the two shorter parallel sides. One of the grooves carries the étalon, and the other a counter-

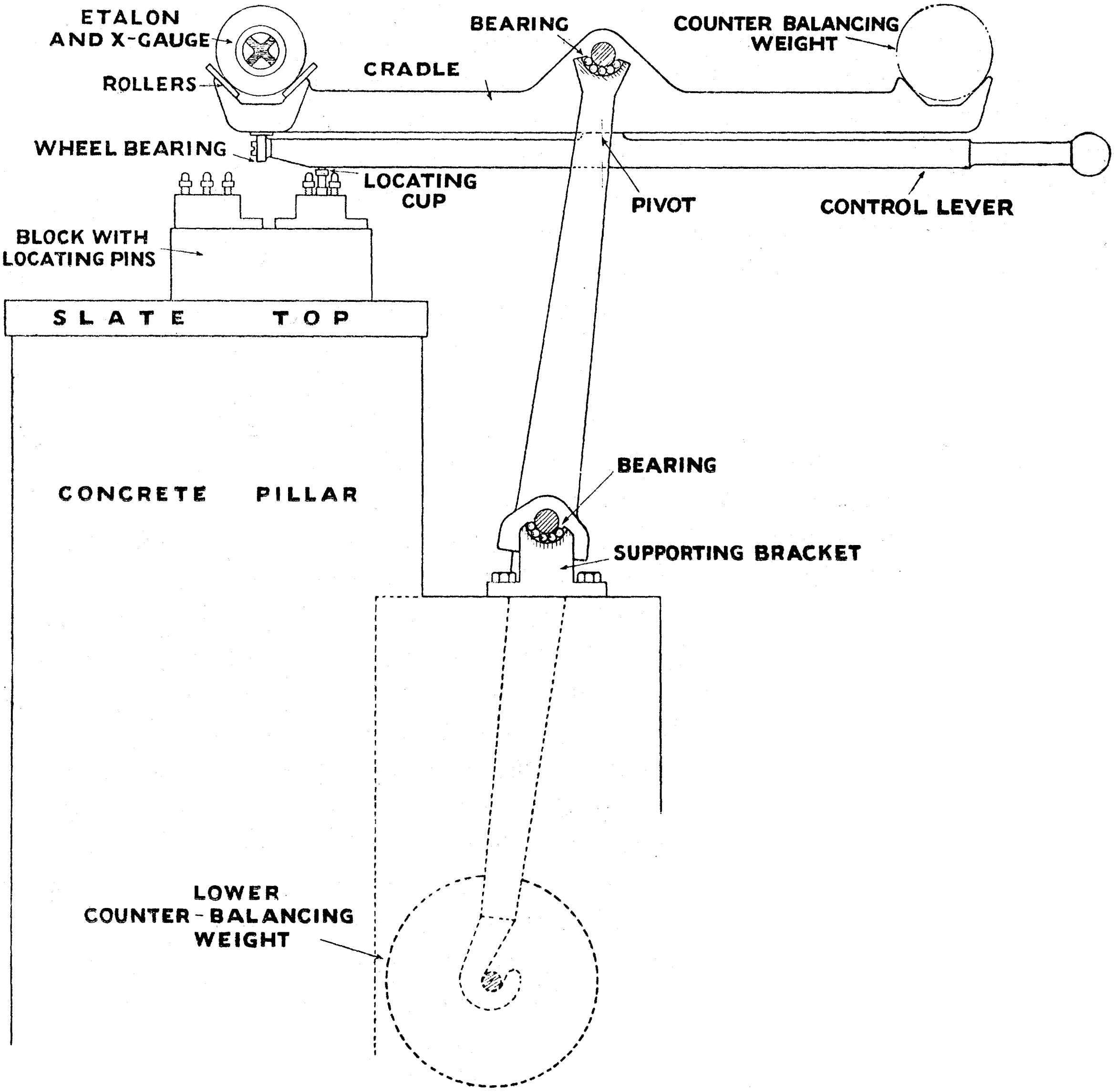

FIG. 7.—Supporting mechanism for longest étalon.

balancing weight. The flanges on the long étalon do not rest directly in the groove. One of them is supported on a pair of steel rollers lying in the groove with their axes in the plane of the flange, and the other is supported on a pair of flat-faced steel blocks, of thickness equal to the diameter of the rollers, also lying in the groove. It was found experimentally that if the étalon flanges were supported directly in the groove the friction between the invar and the light alloy surfaces prevented free axial adjustment of the length of the étalon during changes of temperature. Thus, slight buckling of the étalon was caused and consequently variable errors in the parallelism of its terminal surfaces were produced from time to time. The roller support, being nearly frictionless, enables the étalon to adjust itself axially in the groove during changes of temperature.

At the centres of the longer sides of the cradle are fixed two short co-axial steel cylinders which rest in two low-friction bearings constituted by semi-circular V-grooves containing steel balls, situated on the top of the second casting. This latter is also rectangular in shape and has short horizontal steel cylinders fixed at the centres of its vertical sides which rest in similar low-friction bearings carried by two supporting brackets. These brackets are bolted down to the top of a step on the concrete pillar lying at a lower level than its general surface. Between the two brackets the pillar is cut away sufficiently to allow the movements necessary to give the required displacements of the cradle. The lower counterbalancing weight is adjusted to balance the whole of the cradle, with its load, so that the centre of gravity of the whole mechanism is coincident with the fixed axis of the lower bearings. In this way parallel motion of the somewhat heavy étalon assembly is secured without varying to any appreciable extent the distribution of load on the supporting pillar. This is of particular importance in connection with the use of the apparatus as a comparator (*see* p. 104).

A control lever, pivoted at the centre of the cradle, has at one end a small hardened steel wheel which bears against a hardened steel plate fitted beneath the étalon V-groove, and near this wheel carries an inverted cup which may engage with any one of a series of hemispherical studs or pins situated in various positions on a block bolted down to the top of the concrete pillar. In order that the inverted cup may make definite location on the pins, the counterbalancing weight on the cradle is actually adjusted to a value slightly less than, instead of exactly equal to, that of the étalon assembly.

Altogether, there are eight locating pins on the block, a group of five on the right, and a group of three in line on the left; only three of the group of five are shown in fig. 7, the other two being respectively longer and shorter than the three shown pins, and disposed in line with the central pin, one above and one below the plane of the diagram. The locating pins may be adjusted both vertically and horizontally. When the supporting mechanism is located on the central pin of the group of five, the axis of the long étalon is in line with the axis of the other étalons, but when it is located on any one of the surrounding four, one of the four channels is brought into line. For the determination of the parallel-faced gaps the étalon support is located on the central pin of the group of three at the left of the block. The other two location pins of this group have a special function in connection with the alternative use of this part of the apparatus for optical comparisons of end-gauges, which is described in a subsequent section.

Fig. 24, Plate 8, which is a photograph of the apparatus enclosure with part of the back removed, shows the counterbalancing weight on the rear V-groove, the lower counterbalancing weights and the control lever, which is operated from outside the enclosure when the latter is closed.

(*d*) *Mirrors.*—Five different types of mirror holders are used in the apparatus. Since three of the types occur in the group situated at each end of the longest étalon, fig. 2, it is convenient to describe one of these groups in detail and to indicate the positions of similar types of mirror holders in the general arrangement. Fig. 8 is a plan view of the group of mirrors situated at the left or west end of the longest étalon, the group at the east end being similar in construction but arranged somewhat differently. Each group, consisting of two fully reflecting mirrors and one semi-silvered mirror, is mounted on a heavy cast-iron base carried on three levelling screws which are geometrically located on the slate top of the concrete pillar.

Mirror IX in fig. 8 is a type which requires no rotation, and the same type occupies the positions IV, V, IX and XII in fig. 2. The mirror is a circular glass plate, which in the case of Nos. IX and XII needs to be optically flat over its reflecting surface. It is held in a brass frame by three light phosphor-bronze clips fixed to the front of the frame. The frame has a spherical seating, whose centre of curvature is located at the centre of the reflecting surface, and it is held against two springs by a screw with a spherical head bolted to the back of the holder. The two springs make contact with the back of the frame through two studs arranged 90° apart on its periphery, and opposite each stud there is an adjusting screw, passing through the back of the holder, which can force the frame to move about its spherical seating against the action of the opposing spring. The inclination of the mirror is adjustable, therefore, about two axes at right angles.

The second type of mirror is exampled by mirror VII in fig. 8, and occupies the positions III, VII and VIII in fig. 2. Its adjustment is exactly similar to that of the first type, but it is mounted in a different holder, on a pivot situated at one side of the mirror, which enables it to swivel between two adjustable stops. By this modification the mirror can be moved out of the beam of light when necessary.

The two semi-silvered mirrors arranged at 45° to the axis of the longest étalon, of which mirror X in fig. 8 is an example, constitute the third type of mirror. Since it is necessary to view interference phenomena through these mirrors, the former mounting cannot be used. The semi-silvered glass is mounted in an open frame and adjustment about two axes at right angles is obtained by mounting the frame on a pivot at one spot on its periphery and moving it about this pivot by means of two fine screws, passing through the back of the holder, arranged to act at points on the periphery removed 90° from the pivot.

All the mirror holders shown in fig. 8 are bolted down to the base. The holder for the semi-silvered mirror has a socket and set screw in its base to hold a rod which

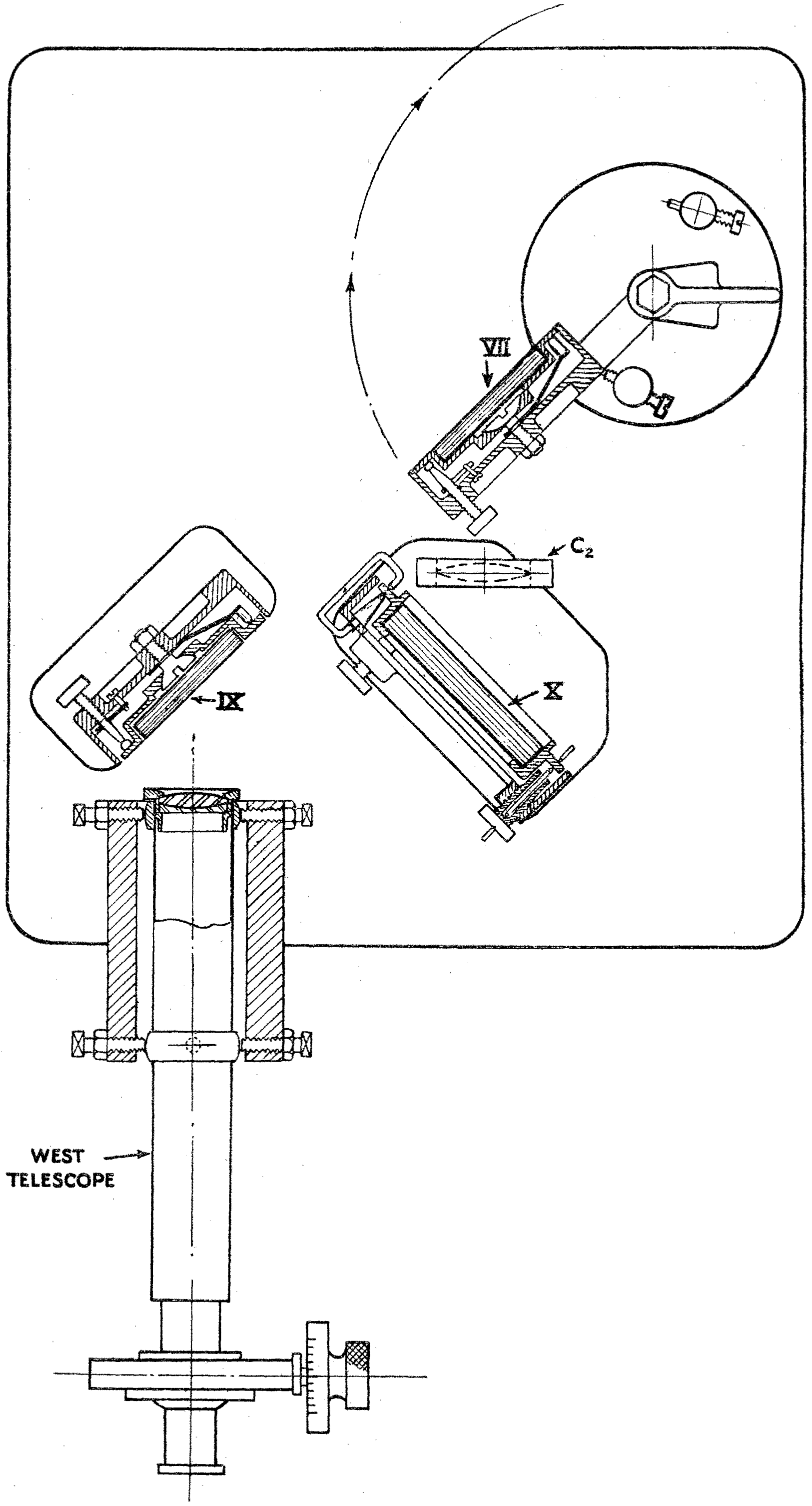

FIG. 8.—Mirror and telescope unit at left end of longest étalon.

carries the converging lens C_2, fig. 2. A separate casting, which is also fixed to the same base, supports the west telescope in a tube having two systems of set-screws for correcting the alignment of the telescope.

A diagram of the fourth type of mirror appears in fig. 9. This type occupies the positions I, II and VI in fig. 2. It consists of a horizontal base of triangular shape with three levelling screws which, in the case of mirrors I and II, are located in three radial V-grooves. A vertical steel spindle is fixed at the centre of the base, about which the mirror holder can rotate, the axis of rotation lying in the plane of the reflecting surface. An arm, integral with the mirror holder, limits the rotation by contact with two adjustable stops. The mirror is held in the holder by three phosphor-bronze clips and adjustment of the inclination of the reflecting surface is made by means of three screws acting against the back of the glass plate.

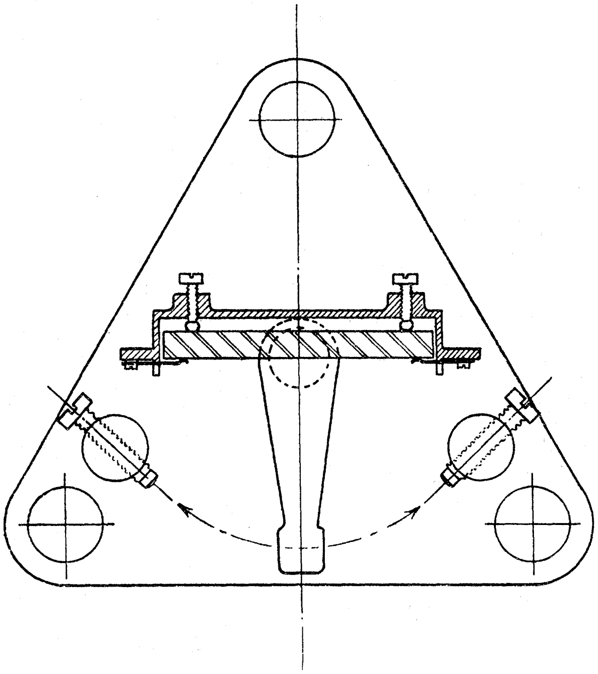

FIG. 9.—Fourth type of mirror.

In the case of the mirror between the first two étalons (Mirror VI, fig. 2), two of the levelling screws rest in locating V-grooves arranged parallel to one another and normal to the line of étalons, and the third rests on a horizontal plane, so that the mirror can be withdrawn from the line of étalons when necessary.

Since the fifth type of mirror and holder is an important part of the optical comparator for end-gauges, which is described in the next section of the paper, a description of this type is more conveniently deferred to that section.

The fully reflecting mirrors are all chemically silvered by a modified Brashear process. They require polishing after leaving the silvering bath, and may be subsequently re-polished many times as and when they become tarnished by exposure to the atmosphere. When not in use, all the reflecting surfaces are protected by tightly fitting brass caps, each of which is lined with a piece of lead acetate paper.

Thin silver deposits, such as are required for the partially reflecting Fabry-Perot and other mirrors, are best obtained by the method of cathodic sputtering. This process is carried out in an evacuated silvering chamber. The high potential discharge is obtained from a transformer delivering 3,000 volts at the secondary terminals, and rectified by a diode valve whose filament is heated by another transformer. A certain amount of smoothing is employed. The usual potential drop across the chamber is 1,000 volts with a current of about 0·02 amp., and the density of the deposit is controlled by the duration of exposure. Exposures range from 6 to 20 minutes according to the reflecting power required by the various mirrors.

(*e*) *End-Gauge Comparator.*—A general outline of the method whereby the X-gauge is compared with the special composite gauge, or with any end-gauge of similar length, has been given in section 2 (*e*). The X-gauge is first introduced between a pair of semi-silvered mirrors independently adjustable into parallelism with the terminal surfaces of the gauge, and the lengths of the gaps between the ends of the gauge and the two mirrors are optically measured. Then the end-gauge to be compared is brought between the mirrors in place of the X-gauge and the gaps are re-determined. The circumstances necessary for successful comparison are that the conditions of support and alignment of the mirrors must be undisturbed during the interchange, which must be performed in such a manner that the axial alignments of the gauges are as near as possible identical when placed in succession between the mirrors.

The two end-gauges to be compared are carried side by side in the V-groove on the supporting cradle otherwise occupied by the longest étalon. A double V-block is placed at each end of the groove and these two blocks support the gauges at their Airy points. Fig. 22, Plate 7, shows the manner of supporting the gauges during the comparison. Since the axes of the gauges are defined by the raised bands at their Airy points, and the ends of the gauges are square to their axes, the ends will be parallel to each other provided that the two bands on each gauge are equal in diameter and that the pair of double V-blocks are accurately constructed and properly located in the long V-groove on the supporting cradle. Any difference in diameter between the two gauges, and any errors in the bands may be compensated by inserting parallel-faced packing pieces, of suitable thicknesses, between the bands and the V-blocks. Roller supports for the gauges, such as are required to reduce the friction between the longest étalon and the V-groove, have not been found necessary, mainly because the surfaces in contact are made of steel resting on hardened steel and also because the points of contact are nearer the axes of the gauges, so that frictional forces at the contacts have less tendency to bend them. As a precaution the points of contact are lubricated with vaseline oil. The same precaution is taken with regard to the contacts between the X-gauge and the longest étalon.

The two mirrors between which the gauges are successively introduced are mounted in adjustable supports at either end of a heavy cast-iron bed, geometrically located on the slate top of the concrete pillar, at a position between the mirror systems IX, X and XI, XII, fig. 2. Thus, the required optical measurements can be made by utilising the same mirrors and telescopes that are employed for determination of the parallel-faced gaps existing between the X-gauge and the terminal plates of the longest étalon.

Fig. 10 is a diagram of one of the mirror supports, which is also seen on the table at the side of the apparatus in fig. 22, Plate 7. The other support is similar in design but arranged in the opposite sense. The mirror is held in an open frame, which is connected by a crossed-strip hinge to an under-carriage. Adjustments about the horizontal axis of the hinge are made by means of a vertical micrometer screw, fixed to the under-carriage and operating, through a ball-ended strut, against the end of an arm integral with the mirror frame. A steel ball, partly embedded in the lower surface of the under-carriage below the mirror, is geometrically located in a socket formed by three other steel balls contained in a ring on the upper surface of the bed. The under-carriage is geometrically supported at two other points on balls between hardened steel planes and Vees in such a manner that it can rotate about a vertical axis through the pivot. The rotation is controlled by a horizontal micrometer screw fixed to the bed. Alternative sockets for the ball-pivot and alternative Vees for the other balls are provided on the bed so that either metre or yard gauges may conveniently be compared. There is no essential objection to using the larger separation of the mirrors for comparisons of both gauge lengths, but the reduction of the parallel-faced gaps to a minimum in each case leads to certain advantages. It would, however, be possible to compare a yard with a metre gauge in one operation if desired. Both mirrors with their supports are completely removable from the bed in order to leave the space between the two pairs of 45° mirrors unencumbered for the operation of measuring the gaps between the ends of the longest étalon and the X-gauge.

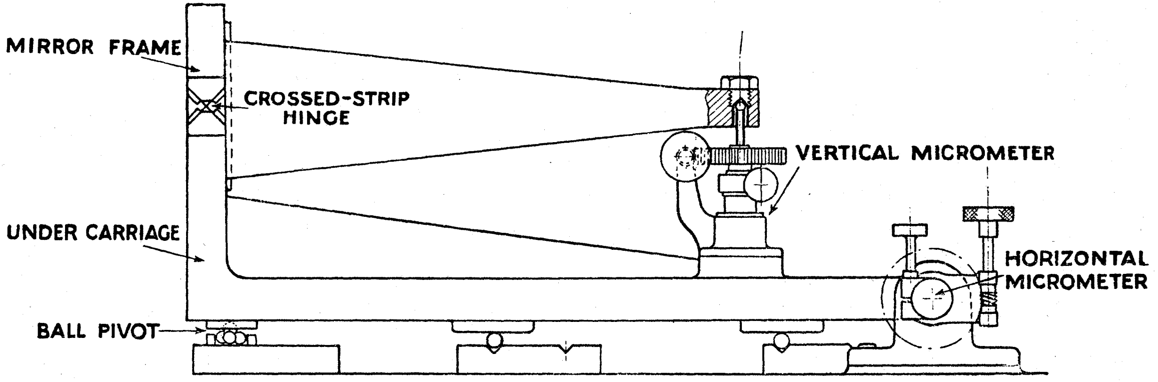

Fig. 10.—Adjustable mirror for end-gauge comparator.

The method of locating the supporting cradle for the gauges has already been described in connection with the alternative use of the cradle as a carrier for the longest étalon and X-gauge. Referring to fig. 7, it will be seen that the pivoting of the control lever of the cradle is only necessary in view of its function of bringing successively the four channels of the longest étalon into the position for optical comparisons with the étalon of intermediate length. In the gauge comparisons, this lever is clamped to the cradle to prevent any longitudinal creep of the cradle which might cause errors in the comparisons. In view of this modification of the locating system the locating cup is replaced by an inverted Vee, with its groove parallel to the axes of the gauges above it. When the locating Vee engages with one of the two outer pins of the group of three situated at the left of the block, fig. 7, the axis of the X-gauge is brought into the line of observation between the two mirror and telescope systems, and when it is engaged with the other pin the axis of the other gauge is brought into line instead.

As already explained, the system of balancing employed in the supporting mechanism for the gauges was specifically designed so that the parallel displacement necessary to bring the two gauges alternately to the same position between the mirrors should have negligible effect on the loading of the concrete pillar, and that consequently the bed carrying the two mirror supports should remain undisturbed, and the separation and alignment of the mirrors therefore remain constant during the interchange. For the same reason the geometrical support of the bed consists of two pairs of Vees in line, and one pair of Vees at right angles thereto, with a steel ball between each pair. Thus the bed is able to adjust itself to changes of temperature without suffering any distortion.

The successful operation of the comparator also requires that the two gauges to be compared should be supported on their bands in the double V-blocks in such a manner that the axes of the gauges are as near as possible parallel. Suppose that the X-gauge is to be compared with the special composite gauge, and that the two gauges are set up in the double Vees. Since the bands on the composite gauge are about 0·6 inch less in diameter than those on the X-gauge, the composite gauge is arranged in the apparatus with packing pieces, consisting of 0·3 inch Johansson block-gauges, placed between its bands and the Vees, in order to bring the axis of the composite gauge to the same height as that of the X-gauge. By means of independent optical and mechanical tests it had previously been established that the ends of the X-gauge were accurately parallel to one another and each accurately square to the axis of the gauge as defined by its bands. The comparator mirrors are then adjusted into parallelism with the ends of the X-gauge, using as a criterion the appearance of the circular interference fringes formed by reflection of monochromatic light at the mirrors and the end surfaces of the gauge. Then the cradle is displaced so that the composite gauge is brought into the line of observation and the appearance of the circular interference fringes at each end is examined. If the axis of the composite gauge is inclined to that of the X-gauge, then the fringes at each end appear to be unsymmetrically focussed by the observing telescopes. The thicknesses of the packing pieces under the composite gauge are in this case suitably altered by units of 0·0001-inch at a time until the asymmetry of the fringes is corrected. Since, in the first place, the circular fringes are visible only when the reflecting surfaces at each end are closely parallel to one another, and it is only the cosine of the very minute angle between the axes of the gauges, possibly remaining after the adjustments described above have been performed, which would affect the accuracy of the comparison, then the condition of accurate alignment of the gauges is satisfactorily realised by means of these adjustments.

The distances between the ends of the gauges and the parallel mirrors are measured in turn, in the same manner as the gaps between the ends of the longest étalon and the X-gauge, using the circular interference fringes, and by subtraction the difference between the lengths of the gauges is found. The case is closed during these comparisons but very precise temperature control is not required, since both gauges are of the same material. The auxiliary platinum thermometers T_1 and T_2 are used to determine the temperature of comparison, and may be seen with their bulbs near the ends of the gauges in fig. 22, Plate 7.

(*f*) *Temperature Control.*—The pillar, on which the main part of the apparatus is mounted, extends from about 6 ft. below to 3 ft. above floor level; it is about 11 ft. long by 2 ft. 6 in. wide, and is completely isolated from the rest of the building. The heat insulating case, about 5 ft. in height, covers that part of the pillar extending above floor level; it is isolated from the pillar and rests on the floor of the room. Fig. 11 is a plan of the enclosure, which shows that it fits closely round the pillar except for a portion in the middle of the north side, where there is a projecting portion necessitated by the shape of the cradle carrying the longest étalon and its counterbalancing weights. This is made as a removable attachment to the main enclosure (*see* fig. 24 Plate 8), and for convenience of reference is known as the "cabin."

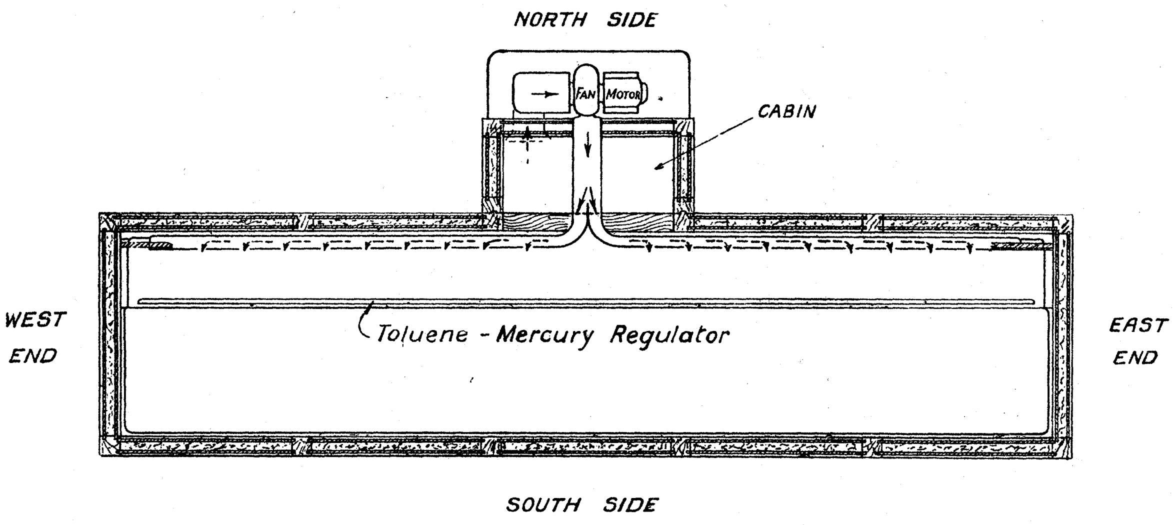

FIG. 11.—Plan of enclosure.

Fig. 12 is a diagram of a typical section through the enclosure and pillar. It will be observed that above floor level there is a change in the material of the pillar from concrete to diatomaceous brick. The latter material possesses much lower heat conductivity than concrete, and was included to prevent undue conduction of heat to earth—in effect, it is an essential part of the heat-insulating enclosure. Any residual loss to earth may be compensated by electrical heater elements, enclosed in seven symmetrically distributed pockets at floor level in the first brick course, in which the individual dissipation of heat is controlled by seven external rheostats, each one in series with the heaters in one pocket. Above this is an uninterrupted course of diatomaceous brick, followed by another course containing fourteen symmetrically distributed pockets of small cross-section for the insertion of thermocouples. There follow above this two continuous courses, another containing fourteen symmetrically distributed air ducts passing right through the pillar, and then a final complete course. The seven courses of diatomaceous brick are surmounted by a concrete beam of the section shown.

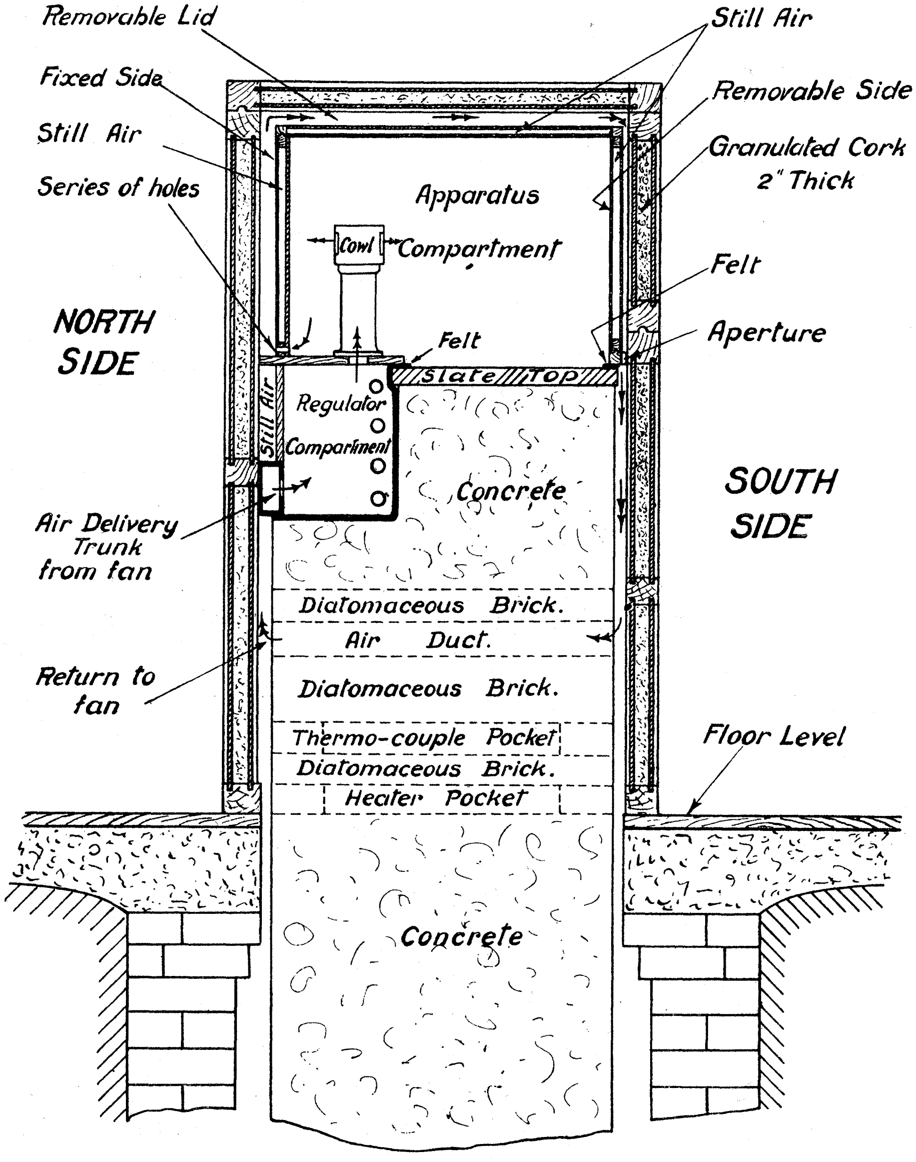

Fig. 12.—Typical section of enclosure.

The enclosure is built of an interlocking system of solid wooden posts and rails filled in with double panels of plywood separated by two inches, the intervening space being packed with granulated cork. The north side, and east and west ends are normally not removable and extend to the full height of the enclosure, but the fixed portion of the south side is limited in height to the level of the slate top of the pillar. The lid and upper portion of the south side are completely removable in eight L-shaped sections, thus giving easy access to the apparatus. All joints were made as air-tight as possible.

The space not occupied by the pillar is divided into two main compartments both of which extend the full length of the enclosure. The division is made by a wooden shelf supported from the north side of the case. A sheet of felt 1 inch thick, covering the exposed concrete faces in the lower compartment, is clamped at one edge between the shelf and the slate top of the pillar, and makes a sufficiently air-tight joint between the two compartments. The lower compartment contains a toluene-mercury regulator extending the full length of the case and supported by suitable brackets off the bottom surface of the shelf. Additional heat insulation is provided for the upper compartment, which contains the apparatus, by an inner enclosure made of two sheets of plywood separated by a nominally still air space $\frac{5}{8}$-inch thick. The north side of this inner enclosure is not removable, but the lid and south side are separately removable in sections of convenient size for handling.

It was not convenient to build the ends of the case in quite the same manner as the sides and top, owing to the number of hand-holes and other openings to be provided for observational and control purposes. Below the level of the pillar top the ends are normal, but above this they consist of two separate inner and outer panels, each similar in construction to the inner sides and top just described. A section through one of the ends is shown in fig. 13.

A diagram of the toluene-mercury regulator is given in fig. 14. The expansion bulb B is of sufficient size to contain the toluene-mercury interface for all temperatures at which the instrument is likely to be used, while the smaller bulb *b* under the capillary is a safety device intended to prevent mercury from emptying out of the sloping connecting tube if a failure of any kind (*e.g.*, in the current supply) should cause the temperature to fall several degrees. The latter feature enables the controlled temperature to be automatically and exactly regained

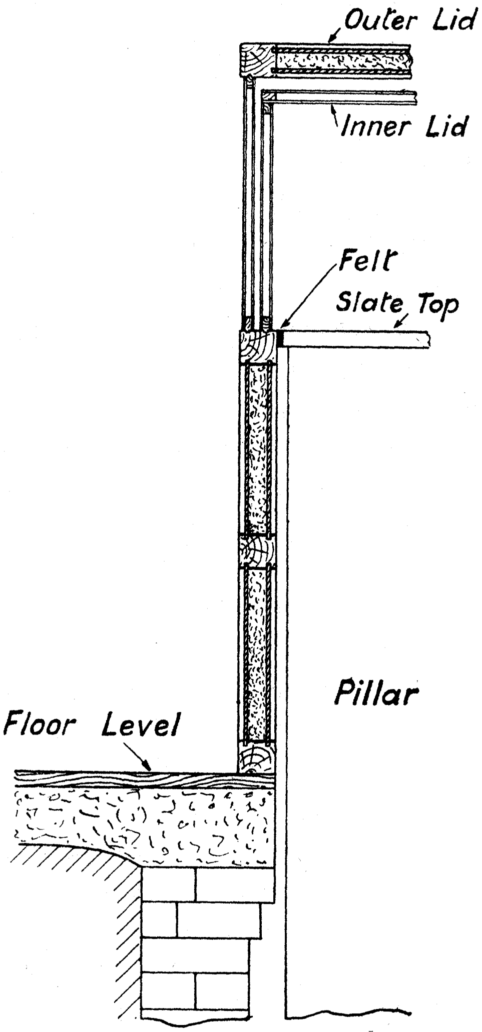

FIG. 13.—Section of end of enclosure.

after the failure is rectified without any interference to the regulator. The joint at J is useful if it is necessary at any time, when the regulator is not working, to clean the capillary portion; it also permits of the substitution of a capillary tube of different bore if it is desired to change the sensitivity. The grid contains a volume of 1,200 ml. of toluene, which in conjunction with the usual capillary tube of 0·9 mm. bore, gives a calculated sensitivity of about 2 mm. per 0·001° C.

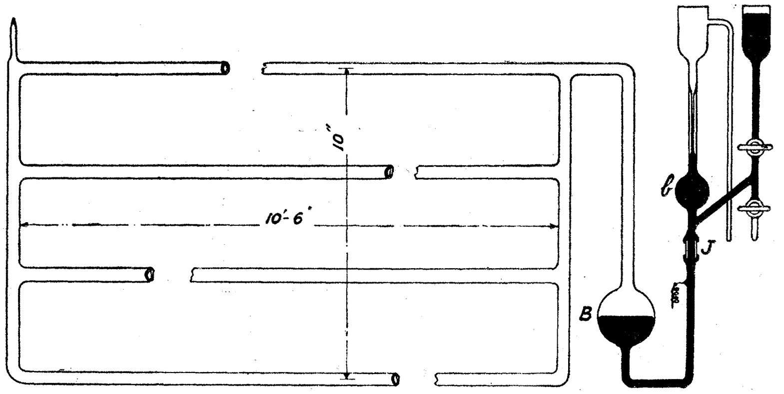

FIG. 14.—Toluene-mercury regulator.

In order to prevent hunting of the regulator a Gouy dipping electrode was provided, driven by a small motor from outside the enclosure, and having a period of oscillation of about 10 seconds. Actual experience showed that, owing to its high sensitivity, the regulator had a natural hunting period of only about 90 seconds under the usual conditions of operation, and with this relatively short period it proved to be immaterial in practice, from the point of view of the constancy of temperature in the enclosure, whether the Gouy control was in operation or not. The current through the regulator contact is limited to 0·006 amp., and this is sufficient to operate an Isenthal relay of the mercury tube type with an internal soft-iron wire connector.

The air inside the enclosure is vigorously circulated by a fan with an enclosed rotor. The fan is placed at the back of the cabin, and is mounted externally. Internally, the cabin is divided into an upper and lower half by a shelf, fig. 24, Plate 8, and the fan draws air through an external wooden trunk from the lower half. Two asbestos-woven heating mats, each capable of dissipating 60 watts, are fixed across the entrance to this trunk, and a suitable switching arrangement enables the mats to be used either singly, in series, or in parallel, so that a total dissipation of 30, 60 or 120 watts may be obtained. The flow of current in the heaters is of course controlled by the toluene-mercury regulator in conjunction with its relay.

From the fan, air passes into another wooden trunk situated in the upper half of the cabin. Since the cabin is completely removable from the main enclosure, this trunk is arranged to make an airtight butt-joint against the entrance to two diverging trunks which lie in the main enclosure at the level of the step in the pillar, figs. 11 and 12, and fig. 24, Plate 8. The two delivery trunks extend the full length of the enclosure and are drilled at intervals with holes through which air passes into the regulator compartment and over the regulator, whence it emerges upwards into the apparatus compartment via eight rotating cowls. Each cowl is frictionlessly pivoted in two self-aligning ball races, and the air issues from two slits at opposite ends of a diameter in such a way that the couple set up by the opposing influences of the jets causes the cowl to rotate at an average speed of 5 revolutions per second. The cowls ensure thorough mixing of air inside the apparatus compartment.

Air leaves the apparatus compartment by a series of holes at the bottom edge of the fixed inner wall at the north side, and passes round the interspace between inner and outer enclosures in the way indicated by arrows in fig. 12. Owing to the extension of apparatus at the centre of the pillar into the cabin, a slightly different air-path is arranged at the centre. Actually the inner wall does not extend across the opening to the cabin, but air is prevented from leaving the apparatus compartment at this place by a double diaphragm of chamois leather which fits round the projecting part of the cradle and across the opening in such a manner as to leave the cradle free to move. Air therefore enters the interspace at the centre through a series of holes at the north edge of the lid of the inner enclosure. Part of the air from the apparatus compartment also flows round the ends of the enclosure through the interspace between the inner and outer walls, fig. 13.

A series of slots at the level of the pillar top on the south side allows the air to proceed from the various parts of the interspace to the space on the south side between the lower portion of the case and the pillar, whence it passes back via the fourteen air ducts in the latter to the north side, and so eventually reaches the starting point at the fan duct entrance in the lower half of the cabin. Circulation is provided in the upper half of the cabin (which is not in direct connection with the enclosure) by means of a hole in the upper side of the air delivery trunk, and two holes at either side of the horizontal shelf.

All power leads are run as far as possible on the outside of the enclosure and then take the shortest route to the required point. Furthermore, all heat-dissipating units, such as motors and rheostats, are mounted externally with radiation shields and liberal air spaces separating them from the walls of the case.

It was found experimentally that the loss of heat by conduction through the ends of the enclosure caused the internal temperature near the ends to be lower than that at the centre. Compensation for this loss is effected by radiation from a 100 watt carbon lamp, fitted with a white hemispherical reflector, set up opposite each end of the enclosure. One of these radiators may be seen in fig. 21, Plate 7, opposite the left end of the enclosure. The appropriate separation of the radiators from the ends was found by trial to be about 18 inches, using as a criterion the attainment of uniformity to within 0·01° C. between the readings of platinum thermometers situated near the ends and centre of the apparatus compartment.

The inner room in which the apparatus is situated is itself thermostatically controlled.

The type of toluene-mercury regulator used is similar in its general character to that already described, though its dimensions are much smaller, the volume of toluene being only about 120 ml., and a fixed electrode is used. The bore of the capillary tube is 2 mm. and under suitable operating conditions the instrument is normally capable of maintaining constancy of room temperature within ±0·1° C. Actually two such regulators are used, one situated at each end of the enclosure on the lid, and each controlling through electro-magnetic relays a battery of four radiator lamps dissipating 1 kw. The lamps are placed behind screens in the two corners of the room most remote from the apparatus. One of the regulators may be seen in position on the end section of the lid of the enclosure in fig. 21, Plate 7. A further kilowatt of heating power, controlled by manual operation only, is used for additional compensation during spells of very cold weather. A fan, situated on the lid of the enclosure between the two regulators, stirs the air in the room. Under controlled conditions the apparatus compartment is maintained at about 1·5° C. above room temperature.

Table II displays the results of a performance test of the whole system of thermal control. Only one thermo-regulator for the room was in use at the time of this test, and it was placed on the centre of the lid of the enclosure, near a mercury-in-glass thermometer, whose readings are given under the column headed "Room Temperature." The column headed "Platinum Thermometer Readings" gives the temperatures indicated by the western (Θ_W) and eastern (Θ_E) halves of the platinum thermometer incorporated in the longest étalon, which was at the centre of the apparatus compartment. The power under control of the large thermo-regulator inside the enclosure was 30 watts, and an equal power was continuously dissipated in the foundation heaters. Steady conditions were achieved after the system had been in operation for one week.

The mean temperatures by Θ_W and Θ_E for the whole test were respectively 24·793° C. ±0·003° C. and 24·793° C. ±0·004° C., where the variations given are the root mean squares of the differences of the individual values from the means. The maximum departures from the mean were +0·006° C. and −0·008° C. for Θ_W and +0·005° C. and −0·010° C. for Θ_E. Apart from the two abnormal readings in the morning of July 18th, all the observed temperatures lie within a range of 0·009° C.

During this performance test on the system of thermal control, an exploration of the spatial uniformity of temperature was made by means of the auxiliary platinum thermometers T_1 and T_2. It has been mentioned earlier that these thermometers are normally suspended from the longitudinal rail over the main line of étalons, fig. 21, Plate 7; they can be moved along this rail by cords operated from outside the enclosure, T_1 covering the western half and T_2 the eastern half of the enclosure. The results of this exploration showed that the readings of T_1 and T_2 were generally within 0·01° C. of the readings of the thermometer Θ, situated at the centre of the enclosure. At one spot, however, it was found that the readings of T_1 were consistently about 0·02° C. higher than those of Θ. It was discovered later that at this place the bulb of T_1 was immediately above and in close proximity to one of the mixing cowls, and it is believed that

TABLE II.—Record of Performance Test of the System of Thermal Control.

Date (1930).	Time.		Room Temperature (°C.).	Platinum Thermometer Readings (°C.).	
	H.	M.		Θ_W.	Θ_E.
July 14	14	35	23·21	24·790	24·791
	16	55	23·21	24·795	24·795
July 15	10	45	23·23	24·795	24·795
	16	55	23·21	24·796	24·796
July 16	10	15	23·21	24·795	24·795
	12	45	23·16	24·798	24·798
	14	50	—	24·798	24·797
	15	30	—	24·797	24·797
	17	05	—	24·799	24·798
July 17	9	45	23·13	24·792	24·792
	11	00	—	24·792	24·792
	12	00	—	24·792	24·792
	14	25	—	24·793	24·794
	15	05	—	24·793	24·794
	15	40	—	24·793	24·793
	16	05	—	24·793	24·793
	17	15	23·22	24·793	24·793
	21	25	—	24·796	24·796
July 18	10	10	23·24	24·785	24·783
	11	45	23·20	24·786	24·784
	12	55	—	24·790	24·790
	15	10	—	24·792	24·792
	17	30	—	24·793	24·792
July 19	9	30	23·17	24·790	24·790

the local high readings were due to excessive frictional heating in the bearings of this particular cowl. The readings of T_1 when it was near the first étalon L_1, in which the basic measurement of length is made, were always within 0·003° C. of those recorded by Θ.

No further extended performance tests have been made, but during the measurements of the X-gauge in vacuum, the provisional results of which are quoted later in the paper, a range of temperature of only 0·005° C. was recorded by Θ in a period of four days, Table IV. p. 126.

A complete description of Θ and the methods adopted for its standardisation and use is given in Appendix II.

(*g*) *Sources of Monochromatic Radiations.*—(i) *Cadmium.* Lamps of the Michelson type have been used for most of the present investigation. The discharge tube itself very closely follows the design first used by MICHELSON,* consisting of two aluminium

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* See MICHELSON and BENOÎT, *loc. cit.*, p. 35.

ring electrodes mounted in glass bulbs connected by a short length of glass tubing of 2·5 mm. bore. A small amount of pure cadmium contained in the lamp is partially vaporised by maintaining the lamp at a sufficiently high temperature in a thermostatically controlled electric furnace. Excitation is performed by a transformer in which the secondary voltage can be adjusted, and the current through the lamp is further controlled by a water-tube resistance in series with the lamp.

Instead of being highly evacuated, as prescribed by MICHELSON, the lamps used in the present work were connected to a large external bulb containing air at about 1 mm. pressure. This modification was first suggested by PÉRARD,* who found that it provided a lamp of much longer life. Experience with the modified lamp shows that this observation is correct, but that the air supply needs to be replenished from time to time. Therefore, a second discharge tube with a stopcock and connector to a vacuum pump is joined to the large external bulb. The second discharge tube has two parallel disc electrodes, and may be used as a pressure indicator by observing the width of the Crooke's dark space when it is excited by a transformer discharge, if it has previously been calibrated by reference to a U-type pressure gauge containing glycerine. A photograph of the complete lamp and furnace is given in fig. 25, Plate 8. The lamp with its external bulb and pressure indicator, which are all supported from the "Mabor" end-plate of the furnace, is shown withdrawn from the furnace.

According to PÉRARD (*loc. cit.*), the most favourable conditions for maximum visibility of interference fringes in the red radiation, at a path difference of 200 mm., are the following: temperature 320° C., pressure 1 mm. of mercury, current 3 milliamperes a.c., with a potential difference between the electrodes from 400 to 600 volts. The current may be varied within large limits without affecting visibility. These conditions have been maintained in the Michelson type lamps used during the present investigation.

Wratten filters Nos. 26 and 55 were used to isolate respectively the red and green radiations of cadmium. The blue radiation was not sufficiently intense for visual work in the present apparatus, but may be used if the interference phenomena are recorded photographically.

A very promising new cadmium lamp is now made by the Studien-Gesellschaft für Elektrische Beleuchtung, Berlin, which is very similar in construction and operation to the sodium lamp† recently made and described by the same firm. It can be very simply excited from the 220 volts a.c. mains and has the advantage of presenting a very large and brilliant source. Experiments made on one of these lamps show that the red line it emits is highly monochromatic, and further investigation of the properties of the new lamp is planned.

(ii) *Krypton.*—A krypton lamp is of great value for certain of the preliminary operations of standardisation, particularly if a monochromator is used to isolate the various radiations. In particular, the strong yellow and green radiations produce visible interference

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* PÉRARD, 'Rev. d'Optique,' vol. 7, p. 10 (1928).

† REGER, 'Z. Instr.Kunde,' vol. 51, p. 472 (1931).

phenomena over path differences exceeding 200 mm. At one time it was impossible to obtain these lamps in this country, and the Laboratory was fortunate in having a batch of these lamps presented to it by the Director of the Reichsanstalt. The lamps are of a modified Geissler type, with a capillary tube source, and are excited by rectified current from a transformer, the potential difference at the electrodes being 400 volts for a current of 20 milliamperes.

(iii) *Mercury.*—Another source of great utility is the 4-feet type of Cooper-Hewitt mercury arc. Although the mercury radiations have fine structure, they have great intensity and can be safely used as auxiliary radiations in the application of the method of coincidences of fractions, if they are applied only to the determination of short path differences.

(iv) *Neon.*—Neon radiations are useful in certain stages of the work, but they also are subject to hyperfine structure. The source used was a Geissler tube containing neon excited by a discharge from a transformer. The radiations cannot be properly isolated with filters and a monochromator was used for this purpose.

(4) *Adjustment and Operation.*

(*a*) *Adjustment.*—The alignment of the sources and their associated lens systems, together with the proper adjustment of the mirrors to direct the beams of light to their correct destinations and the preliminary adjustment of the main telescope, is quite a straightforward procedure and requires no description.

Adjustment of the two systems, each consisting of a converging lens, two mirrors and a telescope, for observing circular interference fringes produced at the gaps between the ends of the longest étalon and the X-gauge was performed in the following manner. First, the semi-silvered mirrors set at 45° to the axis of the étalon, and the converging lenses, were arranged so that a circular patch of monochromatic light, about 1 cm. diameter, illuminated the central region on each terminal face of the gauge. Then the fully reflecting mirror and telescope at each end were adjusted until the centre of the system of concentric rings was seen situated on the intersection of the cross-wires in the eyepiece of the telescope.

The terminal plates of each étalon were next brought into parallelism, using a parallel beam of white light originating from the pinhole situated at the principal focus of the lens of 150 cm. focal length, fig. 2. The main telescope was used for observing the images of the pinhole produced by beams multiply reflected at the semi-silvered surfaces of a given étalon. During these adjustments the straining wires were tightened up with a spanner until these images were coincident with the image produced by the directly transmitted beam.

Reference to Table I (p. 92) will show that L_6 was about 18 μ longer than $3L_5$ when both L_6 and L_5 were unstrained. Since the fringes of superposition in white light are not observable in this case unless the optical path $6L_5$ exceeds $2L_6$ by a quantity not much exceeding 2 μ, it was necessary to reduce the length of L_6 by means of its straining wires, and during this process its length had to be repeatedly measured, as a guide to further adjustment, until its length was within 1 μ of the size at which the fringes of superposition produced by L_5 and L_6 should be visible.

For the adjustment of length of L_6, then, the X-gauge was placed in its normal position along the bore of L_6, and the supporting mechanism was displaced to the position where the gaps at the ends of the gauge could be determined in monochromatic light. Knowing the approximate length of the gauge, as determined by mechanical measurements, the length of the étalon could thus be approximately ascertained as often as desired during the shortening process. A rough check on the parallelism of the terminal plates of the étalon was simultaneously maintained simply by observation of the circular interference fringes produced at each gap, for the fringes were only visible when the reflecting surfaces were reasonably parallel.

Having brought the length of L_6 to within about 1 μ of the required size, the étalon was displaced back into the main line of étalons. It will have been noted that no adjustment has been provided for altering the inclination of the longest étalon. It was arranged during construction that the mechanism was built up so that the axis of the longest étalon was approximately horizontal. This axis is therefore the fundamental basis of alignment; the other étalons and the three observing telescopes are adjusted with reference to it.

Using a Gauss eyepiece, the main telescope was next accurately auto-collimated with respect to the reflecting surfaces of L_6, taking care to ensure that the axes of telescope and étalon were as nearly as possible in the same straight line. Étalon L_5, previously adjusted into approximate parallelism as described above, was then placed in its proper V-groove, and its alignment adjusted by auto-collimation until the reflecting surfaces were normal to the optical axis of the telescope, again taking care to bring the axis of the étalon as near as possible into the fundamental axis.

A search was now made for the fringes of superposition produced by L_5 and L_6. For this purpose the normal eyepiece was replaced in the main telescope, which was, of course, focussed for parallel light, and a beam from the unrestricted source of white light was directed down one of the channels of L_6 and through L_5. The straight parallel fringes were then seen on a background of the image of the white light source when L_5 was slightly inclined by the compensating micrometer.

By suitable alternate adjustments to the parallelism and length of the two étalons the fringes were brought to maximum visibility, the relative lengths in the final condition being such that when L_5 was inclined in either sense by the compensating micrometer, the central white fringe moved towards the horizontal cross-wire, and presented a suitable aspect for precise setting to the wire. In this condition the length of L_6 was slightly less than three times the length of L_5.

With regard to the setting of the central fringe to a wire, it was found that greater accuracy was obtainable if this was first identified in white light, and the final setting then made in red light, interposing a Wratten filter, No. 26, in the beam from the source. Since the central fringe is achromatic, no error was introduced by this modification. The increased clarity of the fringes in red light was due partly to the confinement of the beam to a narrow spectral range and partly to the superior reflecting power of silver in red light.

The final adjustment for parallelism of L_6 was made by setting the central fringe to the cross-wire for one of the channels and observing the behaviour of the fringes as the other three channels were compared in turn with L_5. Slight adjustments were then made as required until the four channels were all equal to within $0 \cdot 1$ to $0 \cdot 2$ of a wave-length, the changes being followed throughout by equivalent adjustments to L_5 to improve its parallelism and to maintain the fringes of superposition at maximum visibility.

The étalon L_1 was then similarly aligned, and adjusted for parallelism and length, by comparison with L_5, until the fringes of superposition between L_1 and L_5 were at maximum visibility and the length $4L_1$ was slightly less than L_5.

It is important that the process of étalon adjustments should be carried out in this order. If it were performed in the opposite sense, starting with L_1 instead of L_6, then any residual error in parallelism of L_1 would be repeated in L_6, but with about 12 times the original magnitude. By adjusting L_6 first, so that its error of parallelism is as small as possible, the errors in L_5 and L_1 are also reduced to a minimum.

It was also found of great advantage to make the étalon adjustments at the same temperature at which the apparatus was to be maintained and controlled when enclosed. Therefore it became a general rule to heat up the room temporarily to this temperature while adjusting the étalons, so that when the case was closed for the achievement of steady conditions, any alteration in the étalon adjustments, due to temperature changes, was reduced to a minimum.

The results of the étalon adjustments may be summarised as follows. The two reflecting surfaces on each étalon were parallel, the axes of L_1 and L_6 were in the same straight line as the optical axis of the main telescope, and the relative lengths of the three étalons were suitable for optical comparisons by means of the fringes of superposition in white light, compensation being obtainable by inclining the axis of L_5 slightly in a vertical plane containing the optical axis of the telescope. At the same time, of course, when the étalon L_1 was illuminated with a convergent beam of monochromatic light, the system of concentric bright interference rings on a dark background, which serve for the direct optical measurement of L_1, could be viewed in the transmitted light with the centre of the system coinciding with the intersection of the cross-wires in the telescope eyepiece. Furthermore, when either one of the gaps between the ends of the étalon L_6 and the X-gauge was illuminated with a convergent beam of monochromatic light, the system of concentric dark rings on a bright background could be viewed by reflection with the centre of the system coinciding with the intersection of the cross-wires in the eyepiece of the west or east telescope.

The densities of the partially reflecting silver deposits are different for the three étalons. For the first étalon the deposits need to be highly reflecting for the best observations in monochromatic and white light, but, remembering that the intensity of the usual sources of monochromatic light is generally low, so that a reasonable degree of transparency is also necessary, a compromise was made in fixing the reflecting power for these surfaces between 0·6 and 0·7. The second étalon is used only in white light and the reflecting power in this case was between 0·7 and 0·8. The longest étalon requires both high reflecting and high transmitting powers, and another compromise was made by arranging for equal powers of transmission and reflection of about 0·4.

(*b*) *Preliminary Determination of Orders of Interference in Air.*—Before measurement of the X-gauge can be undertaken, it is necessary to make a preliminary determination of the orders of interference, in terms of the red radiation of cadmium, for the first étalon, and for the sum of the two gaps between the terminal faces of the longest étalon and the X-gauge. Having once ascertained the whole numbers of wave-lengths in these orders, the complete orders may readily be redetermined at any time by a simple process which forms part of the regular sequence of operations necessary for a complete measurement of the X-gauge, and the time actually occupied in this measurement is greatly reduced.

The method of deducing the order of interference for an étalon is a well-known procedure. In the case of the first étalon L_1, an approximate value of its length was already known from the mechanical measurements (Table I), and from this the approximate value of the order for the étalon in the red radiation of cadmium was calculated. The true value of the order was then derived by the method of coincidences of fractions from observations in the following radiations, accepting temporarily the wave-lengths stated below :—

λ_1	Cadmium Red	0·6438 4696μ	(BENOÎT, FABRY & PEROT, *loc. cit.*).
λ_2	Krypton Yellow	0·5870 9154μ	(PÉRARD, *loc. cit.*).
λ_3	Krypton Green	0·5570 2892μ	,,
λ_4	Cadmium Green	0·5085 8224μ	,,
λ_5	Krypton Violet	0·4502 3546μ	(HUMPHREYS)*

Lest it be objected that the assumption of these values is, in effect, an assumption of the result which it is the whole object of the work to obtain, it is pointed out that for the purpose of applying the method of coincidences of fractions, which is the only use to which these assumed values are put, it is only the *ratios* of these values which need to be known with high accuracy, a quite moderate accuracy being sufficient as regards their absolute values. And it may be emphasised that as regards the values quoted, other than the first, it is, in fact, the ratios of these to the first, and not their absolute values, which have been the direct subject of highly accurate experimental determinations by previous observers. For the purpose in view an accuracy of 1 part in

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* 'Bur. Stds. J. Res.,' vol. 5, p. 1041 (1930).

5 millions in the values of the ratios would be amply sufficient, and the ratios are known to be established to well within this accuracy. As regards the absolute value for the red radiation of cadmium, this is only required for the ascertainment of a first approximate value of the whole number of such wave-lengths in the double length of the first étalon, which is roughly 260,000. And since any given distribution of excess fractions in the five wave-lengths only recurs at intervals of 102 wave-lengths (taking as a criterion agreement of observed and calculated fractions to within 0·15 of a fringe, whereas in the groups of coincidences actually accepted in the present work agreement was found in all cases to within 0·1 fringe), a knowledge of the absolute value to an accuracy of only 1 part in 10,000 is sufficient to identify with certainty which particular group of coincidences is to be taken. Thus the value finally obtained is entirely independent of the absolute value provisionally assumed, provided only that the latter is not in error by more than this amount. Properly regarded, therefore, the fact that the above assumed values furnish satisfactory coincidences of fractions, which in turn lead to a result in close agreement with the absolute value provisionally assumed for the red radiation of cadmium, constitutes a complete justification of the procedure adopted.

For the determination of its order of interference the first étalon was situated in its normal position in the apparatus, but the eyepiece of the main telescope was removed so that the real image of the circular interference fringes, produced in light transmitted by the étalon, could be focussed on the wide slit of a spectrometer with camera attachment. Diameters of five bright rings in each radiation were measured off the photographic plate (Ilford Soft Gradation Panchromatic) by a travelling microscope and from these the values of the excess fractions were calculated by a least squares method (Rolt and Barrell, *loc. cit.*). The method of coincidences of fractions was then applied to these values, in conjunction with the approximate knowledge of the order of interference for the red line, to determine the true orders of interference for this line and also for the auxiliary lines. Under normal atmospheric conditions the order for the red line is not likely to vary from time to time by more than ± 2 waves. Subsequent determinations of the order then resolve themselves into measurements of the excess fractions for the red and green radiations of cadmium alone, whereby the orders of interference for both radiations may be easily and certainly identified.

Similar observations were carried out on the gaps existing at the ends of the gauge, but here the sizes of the individual gaps were unknown. Fortunately, an approximate knowledge of the total difference in length between the étalon and the X-gauge was available from mechanical measurements, and this could be applied to the determination of the orders of interference if the excess fractions at each end were added together. Some slight uncertainty might, however, attach to the orders derived in this manner owing to the combined effect at the two gaps of the difference in dispersion of phase change which occurs when light of different wave-lengths is reflected at surfaces of steel and semi-silvered glass or quartz respectively. It was therefore desirable to obtain a check by evaluating separately the orders for each gap. The length of each gap was, of course, only known very approximately in the first instance, but Pérard* has described a method whereby the exact orders of interference may be derived for an étalon whose length is known only to 1 or 2 mm. The method depends on the observation of interference fringes produced by a number of close pairs of lines well distributed in the spectrum, and was applied in the present work by using the following radiations of the spectra of cadmium and neon :—

λ_1	Cadmium	0·6438 4696 μ	(Benoît, Fabry and Perot, *loc. cit.*)
λ_2	Neon	0·6402 245 μ	(Burns, Meggers and Merrill)†
λ_3	Neon	0·6143 062 μ	,, ,, ,,
λ_4	Neon	0·6096 163 μ	,, ,, ,,
λ_5	Neon	0·6074 338 μ	,, ,, ,,
λ_6	Neon	0·5881 895 μ	,, ,, ,,
λ_7	Neon	0·5852 488 μ	,, ,, ,,
λ_8	Cadmium	0·5085 8224 μ	(Pérard, *loc. cit.*).

Using this series of radiations, the orders for the cadmium red and for the auxiliary radiations could be exactly determined for each gap separately if its length was initially known to within about 2 mm. A comparison of the results obtained in this way for the individual gaps with the result calculated independently for the sum of the two gaps showed that the effect of dispersion of phase-change was negligible in these measurements.

Just as for the first étalon, subsequent determinations of the orders of interference of the sum of the gaps for the cadmium red and green radiations can be made by measurement of the excess fractions in these radiations alone.

(*c*) *Preliminary Determination of Orders of Interference in Vacuum.*—The orders of interference for the first étalon in air were first checked by the method described in the previous section, namely, by measurement of the excess fractions for the red and green radiations of cadmium. Then the étalon was evacuated and the excess fraction for the red line only measured again. To determine the change in the integral part of the order of interference, air was allowed to leak slowly back into the étalon through a fine capillary tube and a count was made of the number of complete rings which consecutively appeared at the centre of the interference ring pattern during the filling of the étalon. The integral part of the order of interference in vacuum was then obtained by subtracting the number of counted rings from the integral part of the order of interference in air which, together with the fractional part already measured, gave the true order in vacuum. Similar observations were made in the green radiation of cadmium.

The same process might also be applied to the sum of the end gaps, but in the case of the metre X-gauge, since the gaps were small, and the mechanical reduction in their total length due to evacuation was less than 1 μ, the approximate orders of interference

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* 'Trav. Bur. int. Pds. Mes.,' vol. 18, p. 34 (1929).

† 'Bull. Bur. Stand.,' vol. 14, p. 765 (1918–1919).

for the two radiations in vacuum could more easily and quite safely be derived by direct calculation from those already determined in air. An approximate knowledge only of the refractive index of air was sufficient for this calculation. The exact orders for the cadmium red and green radiations in vacuum were then obtained from the observed fractions in vacuum by the application of the method of coincidences, using the ratio of the wave-lengths for these radiations in vacuum already ascertained with precision by the measurements made in the first étalon.

Some difficulties were experienced at first in the work in vacuum. It was found that the thin silver films on the terminal plates of the étalons rapidly tarnished and became useless after a few hours' exposure to the vacuum. The effect was at first believed to be due to sulphurous vapours given up by rubber tubes connecting the étalons to the vacuum pipe line. Accordingly, lengths of Tombach flexible metal tubing were substituted for the rubber. No noticeable improvement resulted from this modification and after many experiments it appeared that the tarnishing was due to the action of mercury vapour which diffused along the steel pipe line connecting a mercury vapour pump to the étalons. Considering that the 1-inch bore pipe was some 30 feet long, it was rather surprising that the vapour travelled such a distance at ordinary room temperatures. The use of traps was not felt to be a satisfactory solution to the difficulty and so, since a very high vacuum was not required, the mercury vapour lamp was abandoned and evacuation was carried out with a Hyvac oil pump only. Even then, some trouble was still experienced which was attributed to the presence of mercury vapour already absorbed in the wall of the pipe line. The difficulty was finally overcome by lining the étalon tubes with thin sheets of mica silvered by the sputtering process, so that the small amount of mercury vapour still present in the system was prevented, by absorption on the silvered mica, from attacking the important reflecting surfaces of the étalons. At the same time a piece of lead acetate paper was placed in each étalon to absorb any sulphurous vapours which might still exist in the evacuated system. There is evidence to suggest that, as time goes on, the need for these safety devices will become progressively less necessary now that the original sources of the trouble have been removed.

(*d*) *Sequence of Operations.*—When the adjustments and preliminary measurements have been satisfactorily completed, the apparatus is closed, and the thermo-regulating system inside the enclosure is set into operation to maintain the temperature at the desired value (closely approximating to that which existed in the room during the preliminary work). At the same time the room temperature is lowered to a predetermined value, below the enclosure temperature, at which it is known that steady conditions can be properly maintained.

A complete set of observations necessary for standardisation of an X-gauge is obtained in about 75 minutes and requires the services of two observers. One of the latter is engaged solely in making the various observations, while the other records these observations and manipulates the light sources, mirrors and étalons.

First, the three platinum resistance thermometers are read. Then the diameters of five rings in the red and green radiations of cadmium are measured, for the first étalon, in terms of divisions of the eyepiece micrometer fitted to the main telescope.

After this follows the comparison of the first and second étalons, in which the quantity to be measured is the angle through which the axis of the second étalon must be inclined in a vertical plane, with respect to the optical axis of the telescope, in order to bring the central white fringe of superposition into coincidence with the horizontal cross-wire of the telescope. The inclination is first performed in one sense, the central fringe being identified in white light, and the final setting made in red light, as previously explained. The compensating micrometer is then read, and a similar reading is obtained by inclining the second étalon in the opposite sense. Half the difference between the two readings gives the angle, in terms of a fraction of a radian, through which the second étalon has to be inclined from the axis in order to achieve compensation.

In the same manner the longest étalon is compared with the second étalon, each one of the four channels in the former being separately compared with the latter. This requires eight settings of the compensating micrometer.

The longest étalon is then displaced to the position where the gaps at the end of the gauge are determined. As the fringes are observed in light reflected from the gaps, measurements of the diameters of three or more dark rings on a bright background are made in turn by means of the east and west telescopes, in the red and green radiations of cadmium; observations in each radiation are made successively, first at one telescope and then at the other.

At this stage the platinum thermometers are read again, and the series of observations already described is then repeated in the reverse order. A further reading of the thermometers completes the cycle of observations if the measurements are being made in vacuum. In this way every observation concerned in the complete determination is repeated at such intervals of time that all mean readings used in the final calculations relate to the same mean time.

For work in air additional observations have to be made, at suitable intervals during the course of the optical work, of the pressure, temperature, humidity and carbonic acid gas content of the air contained in the étalons and connecting pipe line.

The method of calculating the value of the X-gauge in terms of the red radiation of cadmium is as follows. Excess fractions for the red and green radiations of cadmium in the first étalon are calculated from the measured ring diameters. The mean excess fraction for each radiation is then used for the purpose of deriving the order of interference for the red line by the method of coincidences. The product of this order multiplied by two gives the number of red waves contained in $4L_1$, or four times the optical length of the first étalon.

Let the difference between the two readings of the compensating micrometer for the comparison of the first and second étalons be 2α. By analogy with equation (1) page 81:

$$L_5 \cos \alpha = 4L_1$$

Therefore, since α is small :—

$$L_5 = 4L_1 (1 + \alpha^2/2).$$

Thus the value of L_5 is obtained in terms of the red radiation, and the product $3L_5$ is formed.

If the differences between the two readings of the compensating micrometer for each of the four channels of L_6 when compared with L_5 are, respectively, $2\beta_1$, $2\beta_2$, $2\beta_3$ and $2\beta_4$, then the mean length of L_6 down the four channels may be calculated from four equations of the following type :—

$$L_6 = 3L_5 (1 - \beta_1^2/2).$$

The mean value for the four channels is obtained and a small correction has then to be applied to obtain the axial length of L_6. This correction is determined by a separate experiment described in the next section of the paper.

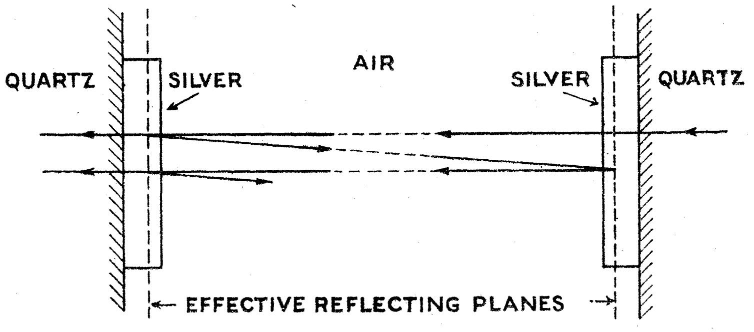
Next, the order of interference for the sum of the gaps at the ends of the gauge is calculated from the measured ring diameters in the manner already described in Section 4(*b*). If half this sum equals G, and X is the optical length of the gauge, then $X = L_6 - G$.

(*e*) *Determination of Corrections in Air.*—The correction to be applied to L_6 to compensate for any difference between the mean length of the four channels and the axial length was determined by previous experiment with the X-gauge removed from the étalon. By optical comparisons with L_5 in the usual manner, the mean length of the four channels of L_6 was correlated with the axial length. This experiment was repeated several times and between each measurement the quartz plates were removed from the étalon and then wrung into position again. The axial length of L_6 was found to be greater than the mean length of the four channels by about $0 \cdot 04\lambda_R$, where λ_R is the wave-length of the red radiation of cadmium. This value could generally be repeated in successive re-wringings of the plates on to the ends of the étalon to within a total range of $0 \cdot 01\lambda_R$.

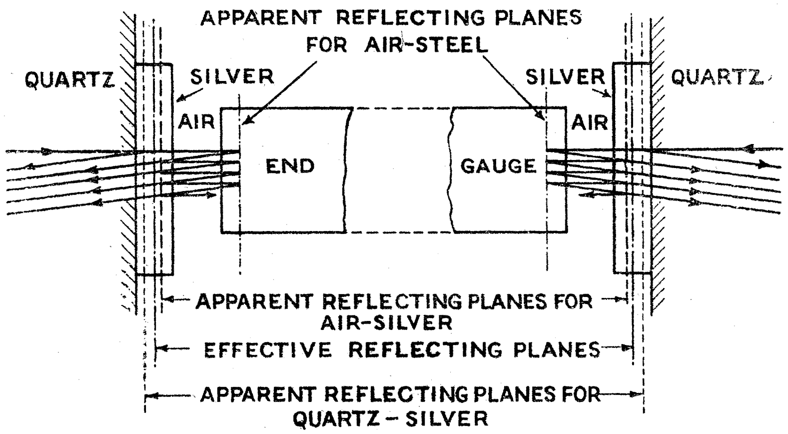
The coefficient of thermal expansion of the X-gauge was measured in a line-standard comparator for a temperature range from 0° C. to 30° C. Fine lines were ruled on one of the webs of the gauge and the usual microscope comparisons were made against a line-standard of known coefficient in the bath of the comparator.

Reference has already been made in Section 2 (*c*) to the effect of the phase change occurring in reflections at a lapped and polished steel surface, which causes the optical length of the gauge to be apparently shorter than the mechanical or practical length. The difference between the mechanical and optical lengths of a steel end-gauge has been obtained from a separate investigation on similarly lapped and polished steel surfaces, a description of which has been published elsewhere (Rolt and Barrell, *loc. cit.*).

Another similar effect of lesser magnitude takes place at the semi-silvered quartz surfaces on the longest étalon. A correction is required for this because the silvered surfaces are used in a somewhat different manner in two stages of the standardisation. Thus, in the optical comparison of L_6 with L_5, the optical path in L_6 is determined by reflections which take place at the air-silver surfaces on its terminal plates, fig. 15 (*a*), while during the determination of (L_6 — X), fig. 15 (*b*), part of the interference effect at each end results from a system of multiple reflections between the air-steel and the air-silver surface, the effective reflecting plane in the latter being the same as in fig. 15 (*a*). But superposed on this there is further interference between this system of rays and the rays directly reflected at the quartz-silver surface, so that the resulting interference phenomenon is somewhat complicated. It may, however, conveniently be treated as corresponding to a path difference equal to twice the distance between the effective reflecting plane in the air-steel surface and an unknown effective reflecting plane in the silver film.



(a) COMPARISON IN WHITE LIGHT



(b) MEASUREMENT OF GAPS (L_6-X)

FIG. 15.—Effective reflecting planes in longest étalon.

Fortunately, direct measurements can be made of the distance between the effective reflecting planes in the silver film in the two conditions of use. For this purpose the quartz plates were removed from the étalon and wrung to the surfaces of two Johansson gauge blocks, of 0·125 inch nominal size and equal to one another to about 1×10^{-6} inch. The blocks were disposed one on either side of the central semi-silvered area so that a Fabry-Perot étalon was formed in which the semi-silvered quartz surfaces acted as reflectors and the gauge blocks acted as separators. The excess fractions in the red and green radiations of cadmium were measured for this étalon in the usual manner, first in transmitted light, where the reflections were similar to those occurring in optical comparisons, fig. 15 (*a*), and then in reflected light, where the reflections at one plate were similar to those occurring in the determination of (L_6 — X), fig. 15 (*b*). The measurements in reflected light were repeated for the other plate. For the actual semi-silvered quartz plates used in the determination of X the combined effect of the apparent displacement of the optical surfaces in the two conditions of measurement amounted to $0\cdot04_4\lambda_R$, and $0\cdot07_4\lambda_G$ respectively in the two cadmium radiations used, and positive corrections of these amounts were therefore applied to the measured values of X (*see* fig. 15).

The last correction to be applied to the value of X is that due to the departure of existing atmospheric conditions from the accepted standard conditions, *i.e.*, dry air at 15° C. and at a pressure of 760 mm. of mercury, containing a normal proportion of 0·03 per cent. of carbonic acid gas. For the preliminary determinations of the metre gauge in air, use has been made of certain data published by PÉRARD*, in which the ratio of the wave-length λ_a, under actual conditions, to the wave-length λ_n, under standard conditions, is given by an expression of the following form :—

$$\frac{\lambda_a}{\lambda_n} = 1 + \mathrm{A} - \mathrm{B} \, . \, \frac{\mathrm{H}}{1 + \alpha t} + \mathrm{C} \, . f,$$

where A and B have values depending on the refractive index of air, and C depends on the difference between the refractive indices of air and water vapour; all of these constants, of course, vary with the wave-length of the radiation. H is the barometric height, reduced to a temperature of 0° C. and to standard gravity, 980·665 cm. per sec. per sec., t°C. is the temperature on the International Scale, f the pressure of water vapour in mm. of mercury, and α has the value of 0·003716.

If for a particular radiation N_a is the number of waves of wave-length λ_a contained in X, then the number of waves N_n of wave-length λ_n under standard conditions is given by :—

$$N_n = N_a \, . \, \frac{\lambda_a}{\lambda_n} \, .$$

(*f*) *Determination of Corrections in Vacuum.*—The correlation between the mean length along the four channels of L_6 and its axial length was made in exactly the same manner as described in the previous section. The axial length of L_6 was found to be on the average $0\cdot00_2\,\lambda_R$ less than the mean length of the four channels. Thus the original value of the correction of $+ 0\cdot04\lambda_R$ in air was practically eliminated by the slight distortion of the quartz plates when the étalon was evacuated. As might perhaps be expected the reproducibility of the value of the correction in vacuum was not quite so good as in air, but the variation still only amounted to less than $\pm 0\cdot01\,\lambda_R$.

The correction for thermal expansion of the X-gauge was, of course, the same as that

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[*] ' Trav. Bur. int. Pds. Mes.,' vol. 18, p. 42 (1929).

in air, and it was also assumed that the corrections for phase change in reflection at steel and semi-silvered quartz surfaces were the same in vacuum as those determined in air.

Also, since the determinations in vacuum were made at a residual pressure of only about 0·01 mm. of mercury, no correction was needed for refractive index, for at this pressure the refractive index of air differs from 1 by a quantity which is negligibly small.

Another correction has, however, to be applied to X to compensate for its elastic expansion in vacuum owing to the removal of the atmospheric pressure. The correction was determined by calculation from a knowledge of the YOUNG's modulus and POISSON's ratio for the gauge, which constants were determined by the usual methods from test pieces cut from each end of the steel bar used for the construction of the yard X-gauge. The metre X-gauge was made of the same brand of steel and was assumed to have the same elastic constants. The calculated change in the length of this gauge when taken from vacuum to one atmosphere amounts to $-0 \cdot 22\mu$.

(5) *Provisional Results.*

Test measurements of a one-metre gauge have been undertaken, both in air and in vacuum, in terms of the red and green radiations of cadmium from a Michelson lamp. It was considered desirable to carry out this preliminary work on the metre length in preference to the yard, in order that the results might be compared with those of earlier observers. A few additional measurements were made in vacuum in terms of the red radiation of cadmium as exhibited by the new German lamp mentioned in section 3 (*g*).

The real object of the preliminary work was simply to gain experience in the adjustment and operation of the apparatus, and for this reason none of the results given below are to be considered as final. During the measurements in air the étalons were left open to the atmosphere of the enclosure, and the full degree of temperature control was not imposed on the apparatus. Consequently, the condition of the air inside the étalons was neither controlled nor capable of accurate determination. Since these results are not to be regarded as final it has not been considered necessary to give the detailed observations from which they have been calculated. They are quoted simply as an indication of the general order of accuracy which the apparatus is capable of giving.

Table III displays the results of these preliminary measurements. λ_R and λ_G refer respectively to the lengths of the waves of the red and green radiations of cadmium. The third column gives the numbers of separate determinations from which the mean values of the wave-numbers for the mechanical length of the metre X-gauge, given in the fifth column, have been calculated. The fourth column gives the probable errors of a *single determination* of X under the various conditions, calculated from the usual formula

$$\text{p.e.} = 0.6745 \sqrt{\frac{\Sigma d^2}{n-1}}$$

where n is the number of determinations and Σd^2 is the sum of the squares of the deviations of the single determinations from the mean. The values of X refer to its mechanical length at 62° F. (16·667° C.); those in air are corrected to the usually accepted standard atmospheric conditions, and those in vacuum are corrected for the small change in length due to the difference of pressure of one atmosphere.

TABLE III.—Provisional Values of λ_R and λ_G.

Radiation and Source.	Condition.	No. of Determinations.	p.e. of a Single Determination.	Wave-Number for X at 62° F.	Wave-Number for 1 Metre.	Wave-Length (1×10^{-6} Metre).
λ_R (Michelson Lamp)	Standard air...	14	±0·13	1,553,165·80	1,553,163·69	0·6438 4714
,, ,,	Vacuum ...	4	±0·05	1,552,736·55	1,552,734·44	0·6440 2513
,, (New Lamp)	Vacuum ...	3	±0·05	1,552,736·58	1,552,734·47	0·6440 2512
λ_G (Michelson Lamp)	Standard air...	14	±0·18	1,966,252·87	1,966,250·21	0·5085 8227
,, ,,	Vacuum ...	4	±0·16	1,965,705·22	1,965,702·56	0·5087 2396

As a matter of interest the value of λ_R in air for the Michelson lamp may be compared with the present internationally accepted value of 0·6438 4696 $\times 10^{-6}$ metre as determined by BENOÎT, FABRY and PEROT in 1906. It has been shown*, in the light of more recent knowledge of the line-standards used during the determinations of BENOÎT, FABRY and PEROT that their value of λ_R should be raised to 0·6438 4703 $\times 10^{-6}$ metre. The discrepancy between this value and the present provisional result is then reduced to less than 1 part in 5,000,000, which is very satisfactory having regard to the total number of steps involved in the two comparisons, and the acknowledged uncertainty in the determination of atmospheric conditions in the present preliminary work.

It will be noted that the p.e. of a single determination of the X-gauge in terms of λ_R was much smaller in vacuum than in air and this was undoubtedly due to the elimination of the uncertain determinations of atmospheric conditions which affected the value in air. For this reason it is believed that the final value for λ_R in vacuum will not be substantially different from that already given. The probable errors of single determinations in terms of λ_G under the two conditions were similar to one another, mainly because the lower visibility of the circular fringes in the first étalon for this radiation, resulting from its imperfect homogeneity, gave rise to errors of measurement relatively greater than those due to other causes.

It can be demonstrated that the p.e. of a single determination of the X-gauge was, in

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* GUILLAUME, "La Création du Bureau International des Poids et Mesures et Son Œuvre," p. 224 (1927).

all cases, approximately 12 times the p.e. of a single determination of the first étalon. Thus the p.e. of a single determination of the length of étalon L_1 in terms of λ_R, calculated from the 14 determinations in air, was $\pm 0 \cdot 009 \lambda_R$, while the p.e. for the X-gauge under the same conditions was $\pm 0 \cdot 13 \lambda_R$. Similarly for the vacuum determinations the p.e. was $\pm 0 \cdot 004 \lambda_R$ for L_1 and $\pm 0 \cdot 05 \lambda_R$ for the X-gauge. Since the length of the X-gauge is approximately 12 times that of L_1 it is evident that the greater part of the experimental error is incurred in the basic measurement of the first étalon, and that the increase of proportionate error due to the subsequent procedure of optical multiplication and the determinations of the gaps is exceedingly small. In support of this statement the following results are presented; they are an extract from observations taken during the vacuum determinations and relate to optical comparisons of the longest étalon L_6 with the étalon of intermediate length L_5 on four consecutive days.

Table IV.—Optical Comparisons of L_5 and L_6.

Date (1931).	Temperature.	Values of $(3L_5 - L_6)$ in Terms of λ_R in Vacuum.				
		Channel (1).	Channel (2).	Channel (3).	Channel (4).	Mean.
August 31	22·225° C.	1·468	1·383	1·523	1·320	1·424
September 1 . . .	$22 \cdot 222_5$	1·474	1·372	1·534	1·330	1·428
September 2 . . .	22·225	1·460	1·381	1·528	1·330	1·425
September 3 . . .	22·227	1·466	1·383	1·534	1·333	1·429

It will be observed that temperature changes during the comparisons were small, and that, although there was an error of parallelism of $0 \cdot 2 \lambda_R$, as evidenced by the differing lengths of the four channels, individual values of a given channel showed no greater variation than $0 \cdot 014 \lambda_R$, while the mean was reproduced each day within a total variation of only $0 \cdot 005 \lambda_R$. Similar reproducibility was obtained in the comparisons of L_1 and L_5.

Provisional values of the refractive index of air under the usually accepted standard conditions (*see* section 4 (*e*)) may be calculated from the determinations of the X-gauge in air and in vacuum. Since the refractive index, *n*, is equal to the ratio of the wave-length in vacuum to the wave-length in air :—

$$n = \frac{\text{Wave-number in air}}{\text{Wave-number in vacuum}}$$

$$= \frac{1{,}553{,}165 \cdot 80}{1{,}552{,}736 \cdot 55} = 1 \cdot 000{,}276{,}45 \text{ for } \lambda_R,$$

$$= \frac{1{,}966{,}252 \cdot 87}{1{,}965{,}705 \cdot 22} = 1 \cdot 000{,}278{,}60 \text{ for } \lambda_G.$$

The refractive index of air for λ_R under the same standard conditions, calculated from

PÉRARD'S data*, is 1·000,276,413 and from the data of MEGGERS and PETERS† 1·000,275,814. For λ_G, PÉRARD'S data gives 1·000,278,722 and MEGGERS and PETERS' 1·000,277,925. Had the data of MEGGERS and PETERS been used instead of that of PÉRARD for calculating the corrections involved in the reduction of the measurements of X from existing atmospheric to standard conditions, the final result for X in air would have been lower by about 1 part in 10^7, so that the value found for the refractive index would also have been lower by about the same amount. This change being only of the same order as the probable errors associated with the present measurements the values of PÉRARD would appear to be the better substantiated by these provisional results.

(6) *Summary and Future Programme.*

An apparatus has been designed and constructed which is capable of ascertaining the length of an end-standard of length to a high degree of accuracy in terms of wave-lengths of light either in air under controlled conditions or in vacuum. It is further capable of being used for accurate determinations of the refractive index of air under various conditions, or for precise intercomparisons of end-standards.

In the course of testing the apparatus provisional values have been obtained for the length of the metre in terms of the wave-length of the cadmium red radiation, as emitted from two distinct sources, which agree well with each other, and with the results of previous observers. Further observations made in vacuum, and concurrent observations with the cadmium green radiation, lead to values of the refractive index of air for these two wave-lengths under standard conditions which agree as well as can be expected with those derived from the data given by PÉRARD, but differ appreciably from those calculated from the data of MEGGERS and PETERS.

It is now proposed to proceed to the definitive determination by several independent observers, and using all precautions, of the lengths of the yard and of the metre in terms of the cadmium red radiation, both in air and in vacuum. This in itself will provide :—

(*a*) the first adequate independent determination of the length of the yard in wave-lengths of light ;
(*b*) the first direct determination of the length of either unit in terms of wave-lengths in vacuum ;
(*c*) confirmation (or otherwise) by comparison with the results of previous observers of the stability of length of the metre ;
(*d*) an incidental new determination of the ratio of the yard to the metre ;
(*e*) values of the refractive index of air for certain wave-lengths under certain specified conditions.

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* 'Trav. Bur. int. Pds. Mes.,' vol. 18, p. 42 (1929).

† 'Bull. Bur. Stand.,' vol. 14, p. 697 (1918–1919).

When these determinations have been completed it is proposed, following the recommendation of the International Committee of Weights and Measures, 1931, to extend the investigation to include determinations of the refractive index, in the visible spectrum, of air under various conditions. At the same time an investigation is contemplated of possible alternative sources of pure monochromatic radiations, and of the conditions of reproducibility for such radiations.

When, and not until, all this information has been obtained it will be possible to consider on its merits the advisability or otherwise of adopting any particular proposal for the future definition of the units of length in terms of the wave-length of light, whether in air or in vacuum.

(7) *Acknowledgments.*

The authors' acknowledgments are due to the Director of the National Physical Laboratory for his active interest in this work, and for permission to publish this description of the new apparatus; to Dr. W. ROSENHAIN, F.R.S., for advice in connection with the chromium plating of the invar étalons; to Mr. F. ADCOCK of the Metallurgy Department of the Laboratory and Mr. J. A. HALL of the Physics Department, for their collaboration in the work, described in the two appendices, upon which the successful operation of the apparatus so much depends; to Mr. F. H. ROLT of the Metrology Department who first suggested the method of adjusting the tubular étalons by means of straining wires, and who gave much other practical assistance and valuable advice; to Mr. H. P. BLOXAM of the Metrology Department, for his assistance in connection with the construction and installation of the apparatus; to Mr. H. E. SMITH of the Engineering Department, for determining the elastic constants of the steel used for making the X-gauge; and to Mr. S. WATTS of the Electricity Department for the calibrations of the standard resistance coils. Also to Mr. A. TURNER and Mr. W. LEE of the Metrology Department, who produced the detailed designs and the working drawings, and the diagrams accompanying this paper; to Messrs. J. SIMMONS, C. KNOYLE, H. K. CORBY and W. G. H. TURL of the Metrology Department for their services in connection with the constructional work; to Mr. W. PINFOLD of the Optics Department for the preparation of the optically flat plates of glass and quartz; to Mr. C. D. LAING of the Physics Department, who constructed the glass-work for the toluene-mercury regulators; to Mr. C. A. HARVEY of the Metallurgy Department for his assistance in connection with the electro-deposition processes; and to Mr. V. W. STANLEY and Miss W. M. BATTERSBY of the Metrology Department for their assistance in the observational work and the computations.

In conclusion, it is a pleasure to record our appreciation of the precision achieved by the Pitter Gauge and Precision Tool Company, in the finishing of the surfaces of the étalons and the metre X-gauge.

APPENDIX I.

The Production of Hard Surfaces on Invar Étalons by Electro-Plating.

By F. ADCOCK, *M.B.E., B.Sc., and* H. BARRELL, *B.Sc., A.R.C.Sc., D.I.C.*

Before electro-plating was commenced each invar tube was machined so that the end surfaces were approximately parallel, within 5μ, and the length was adjusted to be approximately 0·25 mm. shorter than the required finished length. The end surfaces of the longest étalon were not machined until the internal bands mentioned in Section 3 (*b*) (p. 92) had been produced, for it was essential that the end surfaces should be formed closely normal to the axis defined by the internal bands, to which eventually they had to be accurately adjusted. The sharp edges of the surfaces to be plated were rounded off to a radius of about 2·5 mm. to prevent undue accumulation of deposited metal at these regions and to reduce strains which might result in subsequent cracking of the deposits. The terminal surfaces were then ground and afterwards lapped on a flat cast-iron plate, the surface of which was impregnated with abrasive material.

Control of the lengths of the tubes was maintained during the electro-plating and finishing processes by the aid of a series of cylindrical invar gauges of nominal lengths equal to the required finished lengths of the étalons. Having standardised these end-gauges by the usual mechanical methods, the lengths of the tubes could easily be determined at any stage in the work by simple comparator measurements. Since both gauges and tubes were made of invar, with ordinary precautions the influence of temperature changes on their relative lengths could be neglected during these comparisons.

The comparator used during the plating processes was a bar micrometer which enabled comparisons to be performed to an accuracy of 2 to 3μ. After the plating processes were completed the tubes were sent to the Pitter Gauge & Precision Tool Company for the chromium deposits to be lapped and polished to optical flatness and parallelism. The methods used by them for this work are very similar to those developed at the Laboratory for the production of the cylindrical rod type of end-gauge.* Control of lengths during the final operations was maintained by comparisons with the invar end-gauge standards using for this purpose a "Level Comparator."* This instrument allows comparisons to be made to an accuracy of about 0·025μ.

Preparation of Invar Surfaces.—The invar surfaces were cleaned first in benzene and then in hot caustic potash to remove grease, etc. They were then subjected to a "reverse" etching process in which the invar was made the anode in a 30 per cent. sulphuric acid bath. The cathode was made of lead and a potential difference of 6 volts was maintained between anode and cathode for a period of 15 seconds. Afterwards the

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* "Annual Report of the N.P.L." (1921), p. 135.

invar was thoroughly washed in distilled water. The object of the " reverse " etching was to release from the invar any abrasive particles picked up during lapping.

Preliminary Nickel-Plating.—As the bath in which the deposition of copper took place was liable to attack the invar surfaces, it was essential to protect them first with thin films of nickel, which were electrolytically deposited. This was carried out in a bath of the following composition :—

Nickel Sulphate	240 gm.
Boric Acid	30 gm.
Distilled Water	1000 ml.

The anode was a pure nickel plate and a potential difference of 2·1 volts was maintained for a period of about 5 minutes. Before proceeding to deposit copper, the nickel-plated surfaces were washed in distilled water.

Copper-Plating.—The copper-plating was carried out in a bath of the following composition :—

Copper Sulphate	175 gm.
Concentrated Sulphuric Acid	25 ml.
Distilled water	1000 ml.

The anode was a pure copper plate and a potential difference of 0·6 volt was maintained between the electrodes. In all the plating processes the étalon was supported face downwards in the bath and was slowly rotated during the passage of current. The rate of deposition of copper under these circumstances was about 0·017 mm. to 0·018 mm. per hour, and a total thickness of 0·08 mm. to 0·09 mm. was usually deposited on each terminal surface.

The thickness of the copper was subsequently reduced to about 0·05 mm. on each surface by rubbing on the flat surface of a cast-iron lap, taking care to maintain approximate parallelism between the surfaces. The length of each étalon so treated was then about 0·15 mm. less than the intended finished length.

After the copper deposits were lapped, they were cleaned in benzene, dipped momentarily in concentrated nitric acid, washed in distilled water, and then replaced in the copper bath for another ten minutes. This was done to release from the copper any abrasive material it had picked up during lapping, and to prepare it for the satisfactory deposition of nickel. The étalon was then washed in distilled water and placed in the nickel bath.

Second Nickel-Plating.—The nickel bath was of exactly the same composition as that described for the preliminary nickel-plating, but the potential difference was increased to 3·8 volts. The étalon was slowly rotated and frequently brushed to prevent the formation of bubbles on the plated surface. The rate of nickel deposition was about 0·025 mm. per hour, and a total thickness of about 0·08 mm. was usually deposited on each surface.

The nickel deposits were then reduced to about 0·05 mm. thickness on each surface by lapping, taking care to maintain parallelism between the surfaces. At this stage the lengths of the étalons were about 0·05 mm. less than the intended finished lengths.

The lapped nickel surfaces were now cleaned in benzene and then in hot caustic potash, and subjected to the "reverse" etching process described above. The étalon was then thoroughly washed in distilled water and placed in the chromium bath.

Chromium-Plating.—The chromium bath was made up as follows :—

Chromic Acid	300 gm.
Concentrated Sulphuric Acid ..	1·1 ml.
Distilled Water	1000 ml.

The anode was made of lead and was shaped by trial in such a manner as to secure a deposit of approximately uniform thickness. A potential difference of 3·2 volts was maintained between anode and cathode, and the temperature of the bath was kept between 20° C. and 25° C. Usually the current tended to rise during the first 30 minutes of plating. As before, the étalon was rotated and the rate of deposition of chromium was about 0·012 mm. to 0·013 mm. per hour. A total thickness of 0·05 mm. of chromium was deposited on each surface.

In this condition the étalon was about 0·05 mm. longer than the intended finished length. The reduction to correct length was performed by lapping and final polishing so that the surfaces were brought to optical flatness and parallelism. When finished each invar surface had on it the following approximate thicknesses of electrolytically deposited metals, in the order named :—

(i) 0·002 mm. Nickel.
(ii) 0·05 mm. Copper.
(iii) 0·05 mm. Nickel.
(iv) 0·025 mm. Chromium.

APPENDIX II.

Temperature Measurements.

By J. A. HALL, *B.Sc., A.R.C.Sc., D.I.C., and* H. BARRELL, *B.Sc., A.R.C.Sc., D.I.C.*

1. *Introduction.*

All temperatures involved in the length determinations are expressed on the International Temperature Scale, which was formally adopted at the Seventh General Conference of Weights and Measures held at Sèvres in September, 1927. In the temperature region with which the present work is concerned the scale is defined by a resistance thermometer made of platinum wire which satisfies the conditions : $R_{100°}/R_0 \nless 1\cdot390$ and $R_{444\cdot6°}/R_0 \nless 2\cdot645$ and which is calibrated at the freezing and boiling points of water (0° C. and 100° C.) and the boiling point of sulphur (444·60° C.). According to the work of one of us* it is probable that the scale defined by such a thermometer in the

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* HALL, 'Phil. Trans.,' A, vol. 229, p. 1 (1930).

neighbourhood of 20° C. does not differ by more than one or two thousandths of a degree from the thermodynamic Centigrade scale, while, for the purposes of the present work, an error of 0·0025° C. in the measurement of the temperature of the longest étalon (L_6) and the X-gauge contained in it would involve an error of only about 1×10^{-6} inch ($0{\cdot}025\mu$) in the final length measurement.

All measurements of temperature inside the case containing the apparatus have been made with resistance thermometers complying with the International Temperature Scale specifications.

2. *The Platinum Thermometers.*

Since any uncertainty in the temperature of the longest étalon and the X-gauge would prove a greater source of error in the final length measurement than a similar uncertainty in temperature at any other part of the apparatus, special care was taken with the measurement of temperature in this region. One of the thermometers (designated Θ) was actually wound on the longest étalon, while the two auxiliary thermometers T_1 and T_2, which were used to explore the temperature uniformity of the apparatus as a whole, were built on exactly the same lines as those which have already been described.* For the sake of flexibility, however, leads of single 0·029 inch gutta percha covered copper wire were used; and since the thermometers were not intended for use above 100° C. the mica was not de-hydrated. The "bulbs" were wound so as to have a resistance of approximately 50 ohms at 20° C. and are approximately 13 cm. in length. The overall length of each thermometer is about 40 cm.

Somewhat different methods had to be adopted in constructing the main thermometer (Θ) wound on the étalon. Twelve radial slots about 20 cm. in length were milled in the étalon along the outside of the tube on each side of the centre, where the necessary current and potential leads are grouped on an ebonite sleeve, figs. 4 and 16. In these slots are wedged strips of clear ruby mica with V-notches cut in their edges at a spacing of 2·5 mm. These notches carry a bifilar winding of pure platinum wire 0·2 mm. in diameter. The terminal arrangements are shown in detail in fig. 16. The copper strips to which the terminals are attached are sandwiched between two concentric ebonite sleeves screwed in place on the étalon, and are thus insulated from it electrically. It will be noted that the winding is divided into two approximately equal parts (designated Θ_E and Θ_W, corresponding with their normal orientation when in use) and each of these is of about 50 ohms resistance at 20° C. The two sections are joined by a stout copper strip to which the terminal P_3 is attached. The resistance measurements are made by means of a potentiometer (*see* Section 3), the steady current being led in at terminals C_1 and C_2. The three potential terminals P_1, P_2 and P_3 thus allow measurements of the resistance of the whole winding or of either of its two halves to be made independently. It will be seen that a certain amount of copper strip is included with the platinum when the resistance observations are made. The resistance of this

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* HALL, *loc. cit.*, p. 2.

strip, however, would not exceed $1 \cdot 5 \times 10^{-4}$ ohm when half the thermometer (50 ohms) is being measured or 3×10^{-4} ohm when the whole thermometer (100 ohms) is included, and since a resistance change of 4 parts per million is approximately equivalent to a temperature change of 0·001° C. it is evident that the inclusion of this amount of copper in the circuit will have a completely negligible effect.

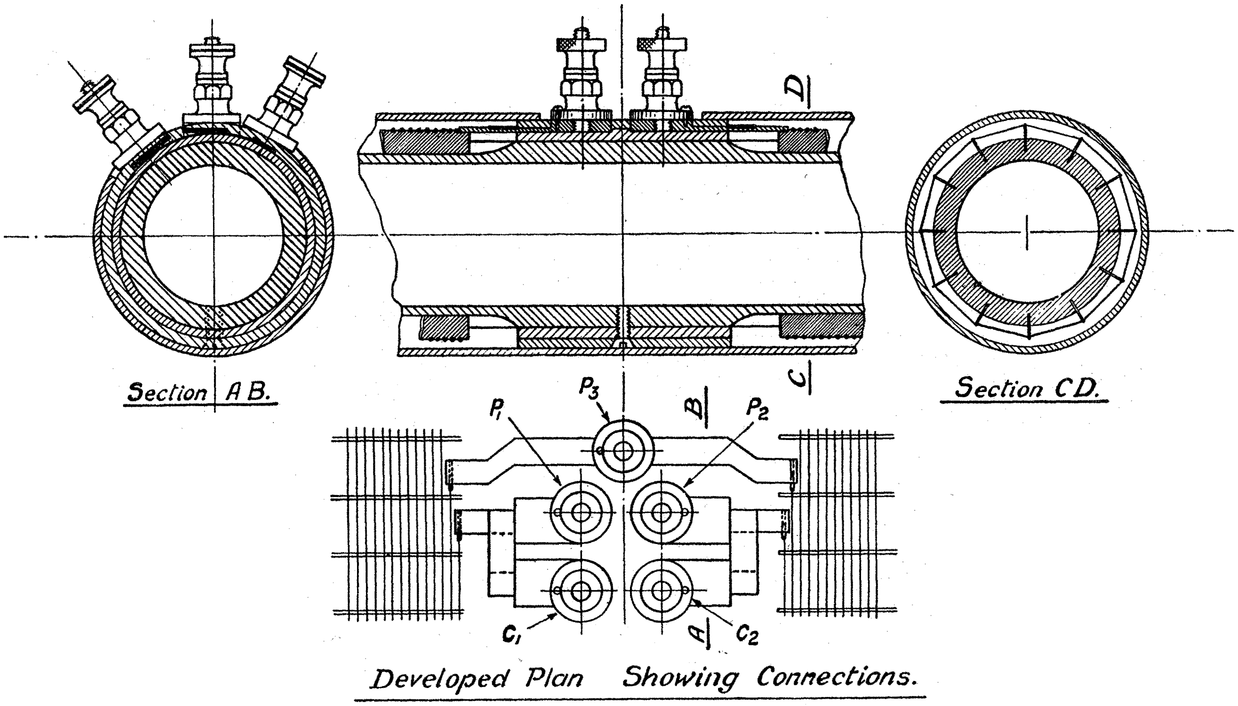

FIG. 16.—Terminal arrangements for resistance thermometer on longest étalon.

The ends of the platinum wire are hard-soldered to very short lengths of copper wire which are in turn soft-soldered into the copper strips. The connecting links between the strips attached to the current and potential terminals are hard-soldered in place. The wire was annealed by heating to 800°/900° C. by passing a current. It was found impracticable to use the ordinary connections (C_1 and C_2) for this purpose as the high voltage applied between consecutive turns of the bifilar winding caused electrical breakdowns when the wire was hot. To avoid this trouble terminals C_1 and C_2, fig. 4, were temporarily connected and the current was led through the two bifilar windings in series by means of platinum leads fused on to the loops at their outer ends. These loops were each arranged to terminate on one of the mica strips and tied in place by short lengths of 0·1 mm. platinum wire fused to them. During the annealing process the étalon was mounted in a sloping position and a strong stream of water led in at the lower end and carried away through a rubber pipe connected at the upper end. When the annealing was complete the brass protecting sheath shown in fig. 4 was slid into place and not subsequently removed.

During the annealing and calibration of the thermometer it was vitally important to protect from possible corrosion the chromium-plated ends of the étalon and the internally ground bands used for supporting the end-gauge, and they were therefore protected by a thick coating of No. 6 anti-sulphuric paint supplied by Messrs. Griffiths Brothers. This was found to be the most satisfactory of four materials tested.

3. *The Resistance Measurements.*

With the exception of certain observations during the calibration of the thermometers, all resistance measurements were made by potentiometer comparison with standard coils. A Tinsley thermo-electric potentiometer modified by the inclusion of a 100-ohm fixed coil was used. It is shown diagrammatically in fig. 17, from which it will be seen that it is possible (without interpolation by galvanometer deflections) to compare two resistances which do not differ by more than about 4 per cent. to an accuracy of 1 part in 250,000. Thus, if the resistance of a thermometer at 20° C. is made equal to that of the standard coil, it is possible to measure temperatures between 10° C. and 30° C. to an accuracy of 0·001° C. Since the potentiometer is only used to compare e.m.f.'s of practically identical values, a very accurate calibration of its coils is not necessary (1 in

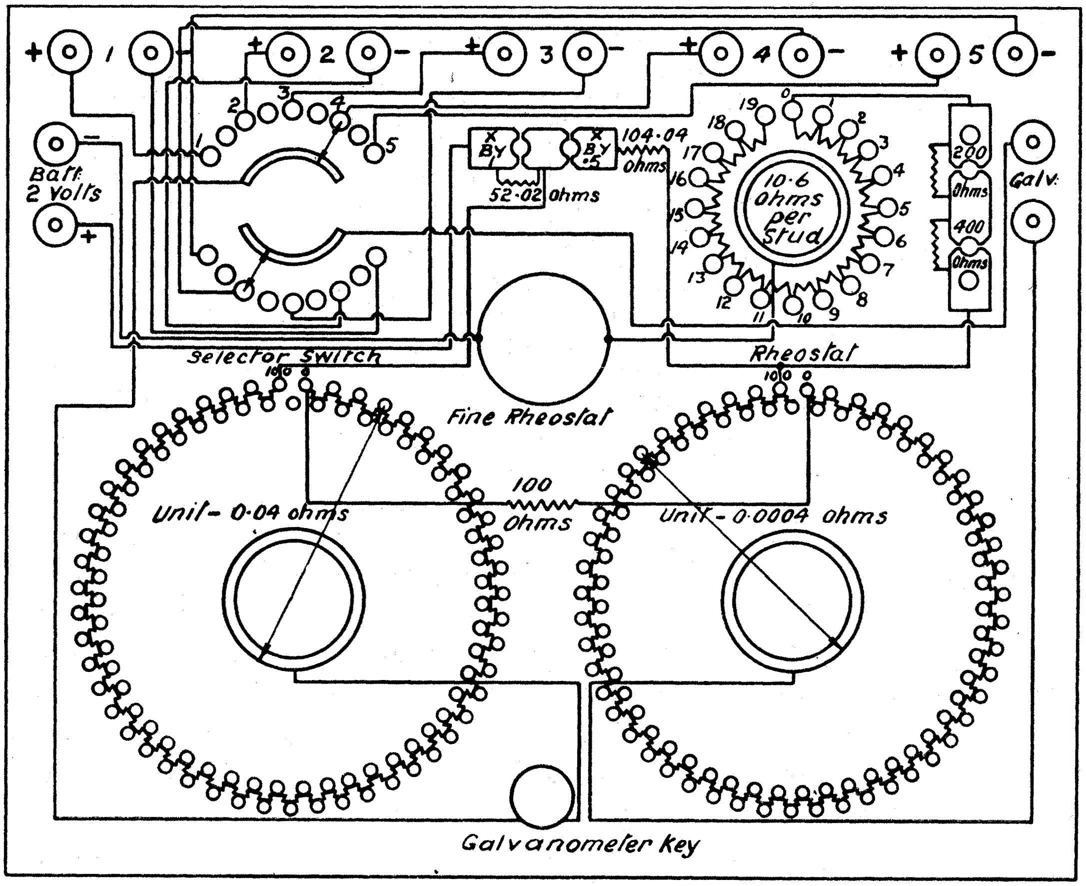

FIG. 17.—Connections of comparative potentiometer.

10,000 is ample) so that a single calibration carried out by the Electrotechnics Department of the Laboratory at the commencement of the work was deemed sufficient. The potentiometer contains a series resistance variable from 0 to 800 ohms, so that any desired p.d. (within wide limits) may be applied to the thermometers, while the use of the 2 : 1 range-changing plug makes it possible to measure both 50-ohm and 100-ohm thermometers without altering the current through either the potentiometer or the thermometer. It should be noted that the range-changing device is only used in the manner described above; it is never used, for example, to compare a resistance of 50 ohms with one of 100 ohms, so that the exactness of its ratio is immaterial. Throughout the work the thermometer current has been maintained at 0·004 amp. In the calibration of T_1 and T_2 observations were also made at the ice point with a current of 0·002 amp. and by this means the heating effect of a current of 0·004 amp. on the thermometer wire under the conditions of calibration was found to be 0·009° C. Similar experiments in the stirred air of the interferometer enclosure indicated a rise of 0·019° C., and a correction of 0·010° C. was therefore applied to the figures given by the calibration. In the case of thermometer Θ, the wire has a cross-section four times as great; moreover, the conditions of calibration correspond more closely to use in air, so that the heating correction is not likely to exceed 0·001° C. and has therefore been neglected.

The connections for the thermometer current and potential leads are shown in fig. 18, which also indicates the way in which the standard resistance coils are arranged to give either 50 ohms or 100 ohms. The two coils are hermetically sealed oil-immersed coils supplied by the Cambridge Instrument Co. and each of 100 ohms nominal value. They are mounted in a bath of paraffin, in which the oil is vigorously stirred by bubbling air.

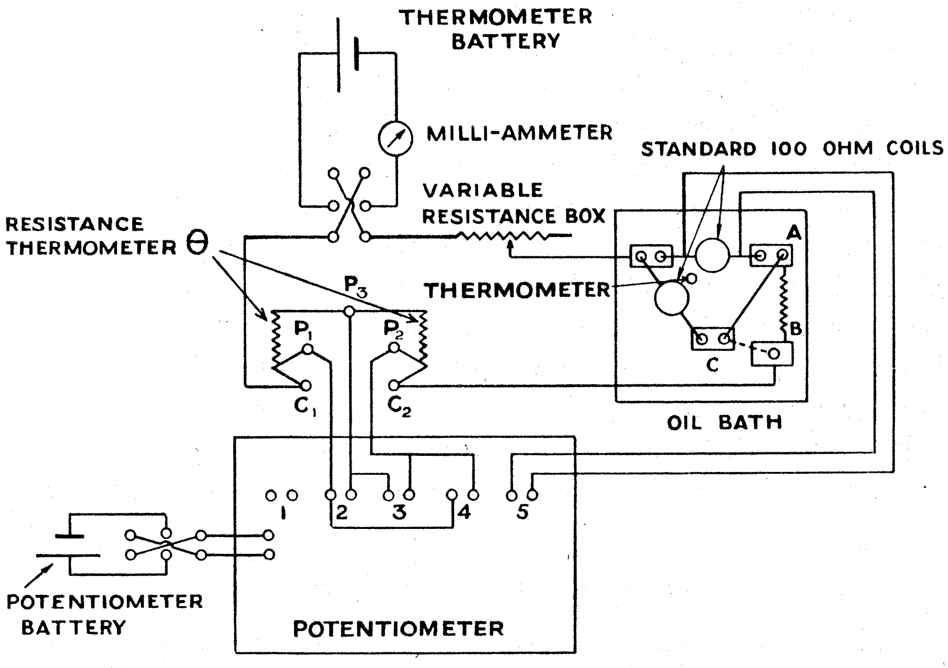

Fig. 18.—Connections for thermometer current and potential leads.

The temperature of the oil is measured by a mercury thermometer divided to 0·1° C. and the temperature coefficients of the coils at 20° C. are just over ten parts in a million per 1° C. The coils have been periodically calibrated by the Electrical Measurements Department of the Laboratory and the values obtained are set out in Table V. When the movable copper link is used to connect the mercury cups A and B only one coil is in circuit, but by transferring the link to cups A and C, as shown in fig. 18, the coils are paralleled and an additional resistance of approximately 50 ohms brought into the circuit for the purpose of keeping the current constant.

TABLE V.—Record of Values of Two Standard Resistances.

Date of Calibration.	Coil L22432 at 20° C.	Coil L22428 at 20° C.
	ohms.	ohms.
October 19, 1928	99·9934	99·9936
April 20, 1929	99·9932	99·9938
November 23, 1929 . . .	99·9944	99·9950
March 1, 1930	99·9942	99·9949
April 4, 1930	99·9945	99·9951
May 11, 1931	99·9945	99·9952

Both dial contacts on the potentiometer are purely potential points so that contact resistance is unimportant, while the effect of parasitic thermo-e.m.f.'s has been eliminated in the customary manner by taking the mean of two readings obtained with the current flowing in opposite directions. For this purpose a special oil-immersed double reversing key, fig. 19, to reverse both thermometer and potentiometer currents was designed, and it has yielded extremely consistent results in use. The contacts are mounted in a stout disc of Keramot, which appears to be free from the troubles attendant on the use of ordinary ebonite under oil. Since an air supply had to be available for stirring the oil bath for the standard coils, the oil in the key has also been stirred, but this is probably an unnecessary refinement. The thermometer current is regulated by means of a resistance box (maximum 1111 ohms variable by 1 ohm) and measured by a 20 milliamp. Weston milliammeter. The galvanometer is a moving coil instrument made by H. Tinsley & Co. It has a coil resistance of 28 ohms, and, under the conditions of use, gives a deflection of about 2·5 mm. for a change in temperature of 0·001° C. on a 50-ohm thermometer.

4. *Calibration of the Platinum Thermometers.*

The two auxiliary thermometers T_1 and T_2 were standardised at intervals at the ice and steam points, using the Smith resistance bridge in the Physics Department*. This bridge is calibrated so as to read in terms of the International ohm as maintained by the Electrical Measurements Department, and a calibration on this bridge should therefore

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* HALL, *loc. cit.*, pp. 5, 12.

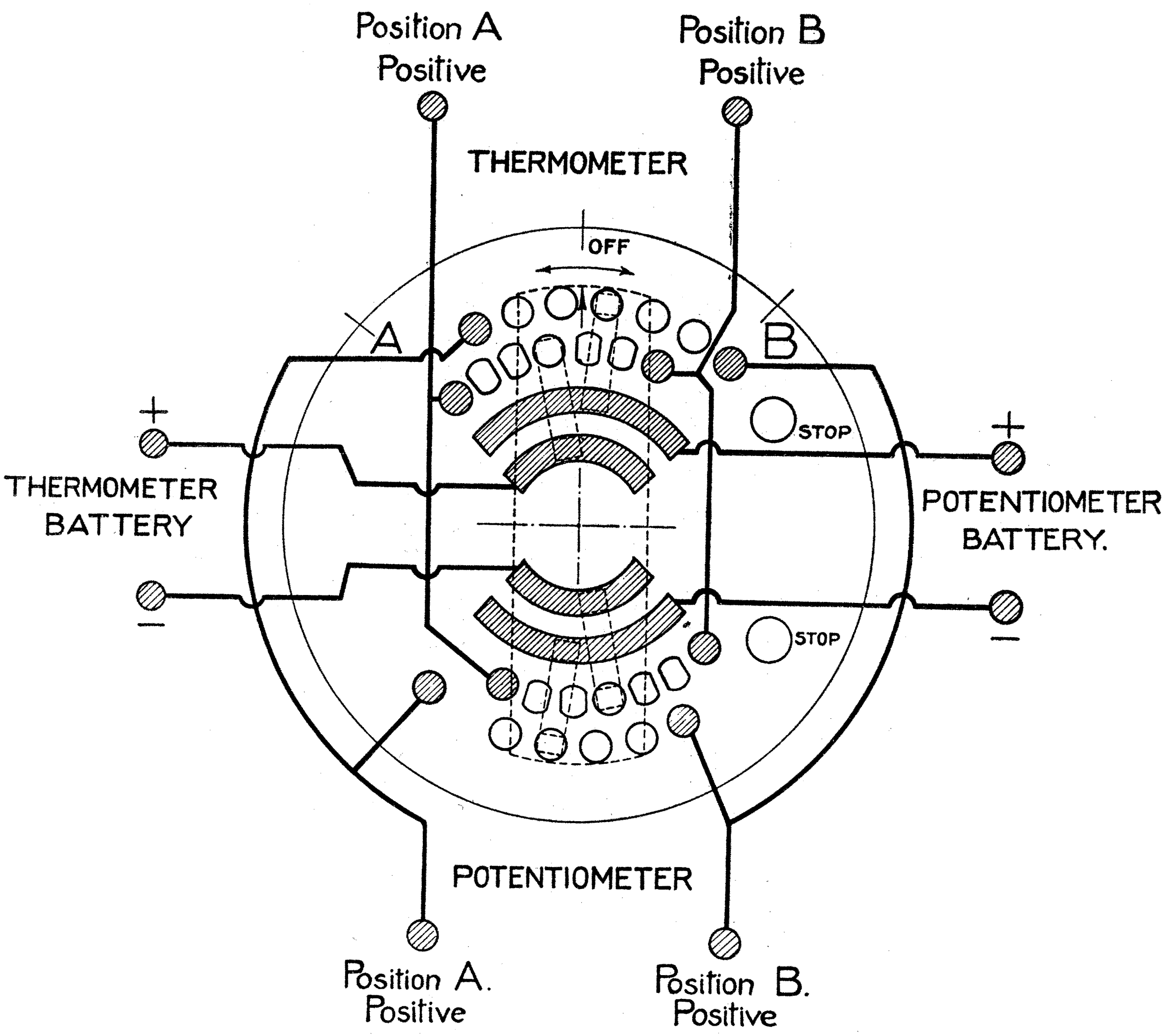

FIG. 19.—Connections of double reversing key.

yield values in accordance with those obtained by comparison with the standard coils used in the potentiometric measurements. As a precautionary measure, however, these coils were measured on the bridge together with T_1 and T_2, and if necessary a small correction (in no case exceeding a few parts in a million) was applied to the figures obtained so as to express them directly in terms of the standard coils. The results obtained in these calibrations are set out in Table VI. As these thermometers were not designed to be heated above 100° C. it was necessary, in order to obtain the calibration at intermediate temperatures, to assume a value for δ based on recent experience. No appreciable error is likely to be introduced by this assumption (*see* p. 144).

As the main resistance thermometer Θ wound on the étalon could not be observed in either ice or steam, reliance had to be placed on comparison with other resistance thermometers for its calibration. To this end the special water bath shown in fig. 20 was constructed.

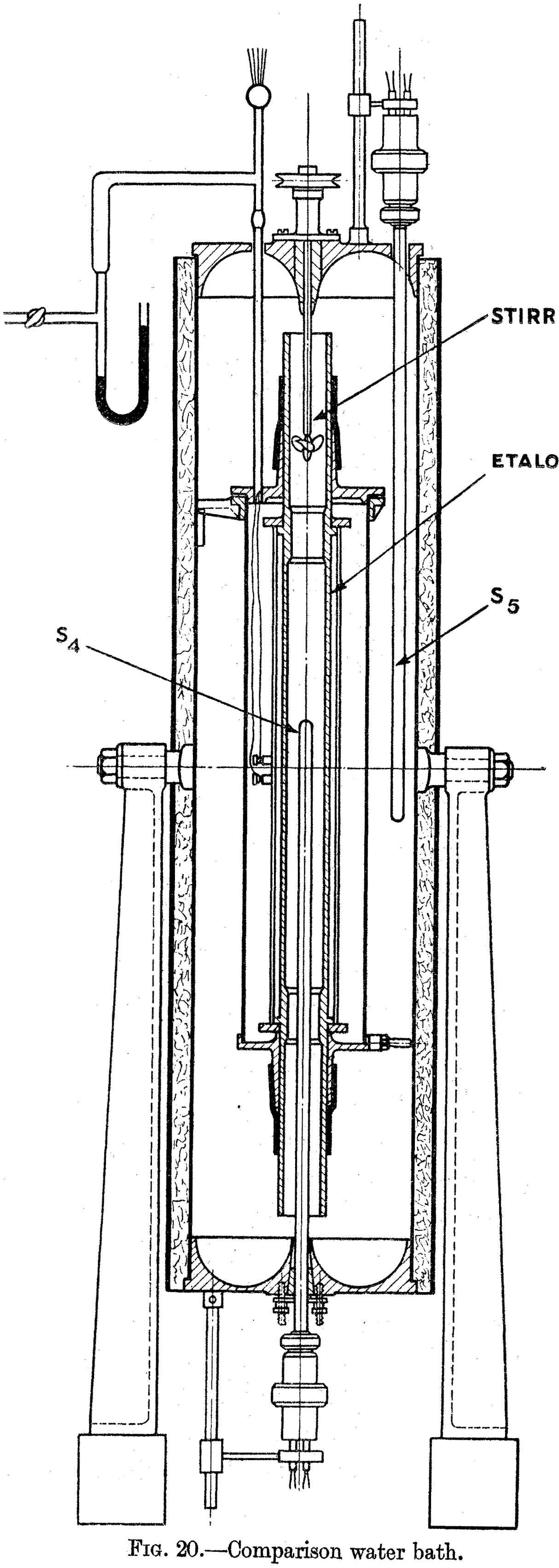

FIG. 20.—Comparison water bath.

TABLE VI.—Constants of thermometers T_1 and T_2.

Date.	T_1		T_2	
	R_0.	R_{100}/R_0.	R_0.	R_{100}/R_0.
4.5.29	45·9722*	1·389716	45·8847*	1·390217
24.5.29	45·9730*	1·389727	45·8856*	1·390223
21.11.29	45·9726	—	45·8852	—
17.5.30	45·9731	1·389711	45·8855	1·390226
13.6.31	45·9730	1·389715	45·8856	1·390217
10.12.31	45·9729	1·389710	45·8855	1·390214

The bath consists of a lagged cylindrical vessel mounted on trunnions. The wall of the vessel is made from brass tube of 10-inch diameter while the ends are brass castings. The lagging consists of 1-inch felt contained in a sheet-metal outer cover. The étalon is placed in a watertight casing, with its ends protruding. This casing is centred in the bath by means of three metal fingers at the bottom and is suspended from the top by three other fingers bearing on a flange. The upper end of the casing is made removable so that the étalon may be slid into place, and it carries a long brass tube which stands clear of the top of the bath and serves as an exit for the five leads from the thermometer. A watertight joint between each protruding end of the étalon and the casing is made by sliding a suitable piece of motor cycle tyre over it and then binding the whole tightly with several layers of pure Parâ rubber insulating tape. A glass T-piece is waxed on to the upper end of the brass exit tube and the leads are led out through a " rose " with five holes blown on its upper end, the joints being made vacuum tight with sealing wax. The side tube is connected to a pump, mercury manometer and drying tube, and a pressure of about 12 cm. of mercury in excess of atmospheric is maintained inside the casing of the étalon. This is sufficient to ensure that, even at the lowest point, any leak will be an outward leak of air and not an inward leak of water.

Two standard platinum thermometers (S.4 and S.5) were normally used in the positions shown. The lower one is introduced through a rubber bung fitting in a hole cut to the same taper and held in place by a brass plate screwed down on to its outer end. The bath is filled with distilled water which is circulated by a screw propeller mounted in ball-bearings and running at a speed of from 1500 to 2000 r.p.m. in the bore of the étalon. Since the diameter of the screw is limited by the fact that the internal diameter of the étalon is only just over 4 cm., care was taken to secure as good conditions as possible for the flow of water. As a result of suggestions made by the William Froude Laboratory, the screw itself was copied from a model of a ship's propeller and the ends of the bath were faired off so as to minimise loss of energy caused by the formation of eddies.

It will be noted that no provision has been made for temperature regulation in the

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* Not corrected in terms of standard coils.

comparison bath itself. The bath, potentiometer and associated apparatus were housed in a small semi-basement room immediately below the room where the Smith Resistance Bridge and the ice bath and hypsometer are installed. The room was made as airtight as possible, the chimney and ventilating outlet being stuffed with cotton waste. The one window, unfortunately, faces south, but the internal wooden shutters were closed and, in addition, a wooden screen was erected outside the window at a distance of about two feet so as to shade the glass completely from the sun at all hours of the day.

Twelve carbon filament radiator lamps, rated at 250 watts each, were fitted on the walls of the room and all were shielded so that no direct radiation from them could reach the comparison bath. By varying the number of these lamps in use the temperature of the comparison bath could be varied from about 13° to 30° C., and the rate of rise or fall of temperature during the observations was generally rather less than 0·001° C. per minute. It was found that the presence of two observers in the room affected the rate of change of temperature of the bath considerably and two substitute observers (100-watt carbon lamps shielded from the bath) were therefore provided. Each observer always switched off his substitute lamp on entering the room, and by this means the conditions could be kept very constant. A 20-watt reading lamp was used for illumination of the potentiometer, and that and the galvanometer scale lamp were left burning continuously.

The stirring was arranged so as to lift the water through the étalon. It was found that with the direction of stirring reversed erratic readings were obtained, probably because of local generation of heat caused by forcing the water into the space just round the bulb of the thermometer inside the étalon. Under the normal conditions of working the uniformity of temperature in the bath was such that, over the 81 observations made, the mean difference between S.4 and S.5 (taking no account of sign) was $0\cdot001_0$° C. and in only one observation was its value as great as 0·003° C. Further, equally good agreement with the central thermometer was obtained when a short thermometer (about 30 cm. immersion) was substituted for the one outside the étalon.

In order to save as much handling of étalon No. 6 as possible a complete preliminary run was made with a similar platinum thermometer wound on a dummy étalon made of mild steel tube. In this way, valuable experience in winding, annealing and calibrating was gained, and all preliminary investigations as to the uniformity of the comparison bath and reproducibility of results were carried out before étalon No. 6 was touched.

The observational procedure was as follows :—One observer (J.A.H.) was stationed at the Smith Bridge with an assistant in the room below (with the comparison bath) to change over the leads from S.4 to S.5 and *vice versa* at a terminal board on the wall in response to bell signals. The other observer (H.B.) was also stationed in this room at the potentiometer. A thermograph was kept running in this room and work was not started unless this showed that a constant temperature had already been maintained for several hours. The subsequent procedure can be followed from the record of a complete observation in Table VII and the attendant calculations on p. 142. Readings on

TABLE VII—Specimen Calibration Observation.

Set XIII—20.3.30. Room temperature 20·6° C. Stirrer 1,700 r.p.m.

Time.	Current Direction.	Standard Coil.			Étalon Thermometer.		Standard Thermometers.		
		Temp. °C.	Nominal Value.	Potentiometer Reading.	Section.	Potentiometer Reading.	Bridge Readings.		Bridge Temperatures (°C.).
							S.4.	S.5.	
H. M.									
11 37								3-3-6-6-7-1	19·68, 20·21, 19·98
11 39							3-1-2-3-4-2·5		
11 41								6-6·5	
11 43							3-6		
11 45								5-9	
11 47	A	20·28	100	1790·5	Θ	5232	2-9·5		19·67, 20·21, 19·96
11 48				1791		5231			
11 49				1791		5230·5		5-2	
11 50	B	20·29		1787·5		5226			
11 51				1788		5225	2-3		
11 52				1788		5224·5			
11 53	B	20·29	50	1834·5	Θ_E	5359		4-5·5	
11 54				1835·5		5358			
11 55				1835·5		5357·5	1-7		
11 56	A	20·28		1844·5		5364			
11 57				1844·5		5364		3-9	19·67, 20·14, 19·93
11 58				1845		5363·5			
11 59	A	20·27	50	1845	Θ_W	5186	1-1		
12 0				1845		5185·5			
12 1				1846		5185		3-2·5	
12 2	B	20·27		1839		5175·5			
12 3				1838·5		5175	0-5		
12 4				1839·5		5174			
12 5	B	20·26	100	1792	Θ	5212·5		2-6	
12 6				1793		5211·5			
12 7				1793		5210·5	2-9-9		19·65, 20·08, 19·91
12 8	A	20·25		1797·5		5214			
12 9				1798		5212·5		2-0	
12 10				1798·5		5212·5			
12 11	A	20·25	50	1847	Θ_E	5349·5	9-3·5		
12 12				1847·5		5349			
12 13				1848		5348·5		1-4	
12 14	B	20·25		1841		5340·5			
12 15				1841·5		5340	8-8		
12 16				1841·5		5338·5			
12 17	B	20·25	50	1841·5	Θ_W	5160·5		0-8	
12 18				1842		5159			
12 19				1842		5158·5	8-2		
12 20	A	20·23		1847·5		5163			
12 21				1847·5		5162		0-1	
12 22		20·21		1848		5162			
12 23							7-6		
12 25								5-9-4	
12 27							6-9·5		19·63, 20·02, 19·85

Sample calculation—group of observations from 11.47 to 11.53 a.m.

Mean time—11.50 a.m.

S.4.
3–1–2–3–2–4·4
$R = 31{\cdot}2175$ ohms*
$R_0 = 28{\cdot}6588$ „
2·5587
F.I./100 = 0·111991
T = 22·847° Pt.

S.5.
3–3–6–6–5–0·2
$R = 33{\cdot}6493$ ohms*
$R_0 = 30{\cdot}8911$ „
2·7582
F.I./100 = 0·120724
T = 22·847° Pt.

Rate of fall = $0{\cdot}001_6$° Pt/min.

Lag correction = + 0·006° Pt.

Corrected temperature at 11.50 a.m. = 22·853° Pt.

Potentiometer Readings.

	L22428 (Nominal 100 ohms)	Θ
	1790·5	5232
	91	31
	91	30·5
	87·5	26
	88	25
	88	24·5
Mean	1789·3	5228·2
Fixed coil (corrected)	250027·5	250027·5
Dials (corrected)	1788·8	5227·8
	251816·3	255255·3

Standard coil L22428 = 99·9955 ohms.

$$\frac{R\,(\Theta)}{99{\cdot}9955} = \frac{255255{\cdot}3}{251816{\cdot}3}$$

Therefore at 11·50 *a.m.*

Temperature = 22·853° Pt. R (Θ) = 101·3611 ohms.

Results of calculations for subsequent groups of observations.

11.56 *a.m.*
Temperature = 22·845° Pt. R (Θ_E) = 50·6965 ohms.
12.2 *p.m.*
Temperature = 22·838° Pt. R (Θ_W) = 50·6601 ohms.
12.8 *p.m.*
Temperature = 22·829° Pt. R (Θ) = 101·3523 ohms.
12.14 *p.m.*
Temperature = 22·822° Pt. R (Θ_E) = 50·6923 ohms.
12.20 *p.m.*
Temperature = 22·814° Pt. R (Θ_W) = 50·6558 ohms.

* For details of bridge corrections see HALL, *loc. cit.*, pp. 9, 21.

thermometers S.4 and S.5 were first taken at two-minute intervals for at least ten minutes in order to make sure that the conditions were good enough for an observation. The signal to start was then given and the two-minute readings continued. The observations on Θ were made in the following order :—Θ against 100 ohms, Θ_E against 50 ohms, Θ_W against 50 ohms, Θ against 100 ohms, Θ_E against 50 ohms, Θ_W against 50 ohms. Each of these six sets of observations comprised six readings on the platinum thermometer and six on the standard coil, the direction of the current being reversed after the first three pairs of readings.

The readings of S.4 and S.5 were plotted independently against time, and each observation on Θ was taken to apply to the mean of the time over which it extended. The observations recorded in any line of the Table commenced at the time shown in the first column, and occupied the succeeding minute. For example, the first set (Θ against 100 ohms) lasted from 11.47 to 11.53 and was held to apply to 11.50 : the temperature at this time was taken from the curves of the resistance of the standard thermometers plotted against time.

The question of the difference in lag between Θ and the standards was dealt with as follows. A straight line connecting the observed resistances of Θ against platinum temperature was computed by the method of least squares and the residuals determined. These residuals were then plotted against the corresponding rates of rise or fall of temperature in the bath and a straight line drawn by the method set out by AWBERY*. This indicated that for a temperature change of 0·001° C. per minute, a correction of 0·004° C. was necessary, *i.e.*, the differential lag coefficient between Θ and the standards was four minutes. The readings were then corrected for lag on this basis, the amount of the correction being in no case greater than 0·008° C., and the average correction, taking no account of sign, only $0{\cdot}002_5$° C. Since the experiments had been arranged to give an almost equal balance between rising and falling temperatures the final uncertainty on account of lag was exceedingly small.

Ice points were observed on both standard thermometers before and after the work and indicated an apparent rise of about 0·002° C. during the four weeks' interval. This could be explained either by experimental error or by a slight drift in the bridge calibration. The observations were first worked out on the latter assumption and it was found that the residuals varied progressively with time. This variation did not appear when the other assumption was adopted and the whole of the observations were therefore related to the mean of the ice points taken before and after the calibration.

The least square equations, independently computed for Θ, Θ_E and Θ_W, are :—

$$\mathrm{R}(\Theta) = 93{\cdot}0324_4 + 0{\cdot}364456_2\,\mathrm{T} \quad (1)$$

$$\mathrm{R}(\Theta_E) = 46{\cdot}5319_7 + 0{\cdot}182307_8\,\mathrm{T} \quad (2)$$

$$\mathrm{R}(\Theta_W) = 46{\cdot}5005_9 + 0{\cdot}182144_8\,\mathrm{T} \quad (3)$$

Table VIII gives a summary of the observations on Θ, Θ_E and Θ_W, together with their

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* 'Proc. Phys. Soc.,' vol. 41, p. 384 (1929).

TABLE VIII.—Resistance-Temperature Relations—Thermometers Θ, Θ_E, Θ_W.

Θ			Θ_E			Θ_W		
Tempera-ture. ° Pt.	Observed Resistance. Ohms.	Residuals (Obs-calc.) × 10^{-4} ohms.	Tempera-ture. ° Pt.	Observed Resistance. Ohms.	Residuals (Obs-calc.) × 10^{-4} ohms.	Tempera-ture. ° Pt.	Observed Resistance. Ohms.	Residuals (Obs-calc.) × 10^{-4} ohms.
13·423	97·9232	− 13	13·427	48·9794	− 4	13·432	48·9466	− 6
13·437	97·9285	− 11	13·443	48·9823	− 5	13·449	48·9497	− 6
14·344	98·2599	− 3	14·348	49·1473	− 5	14·348	49·1139	− 1
14·350	98·2623	0	14·354	49·1487	− 1	14·357	49·1154	− 3
14·360	98·2656	− 4	14·363	49·1503	− 2	14·366	49·1170	− 3
14·972	98·4887	− 3	14·970	49·2610	− 1	14·969	49·2271	0
14·976	98·4905	0	14·975	49·2617	− 4	14·973	49·2278	− 1
15·026	98·5073	− 14	15·021	49·2699	− 5	15·017	49·2352	− 7
15·040	98·5128	− 10	15·035	49·2724	− 6	15·030	49·2378	− 4
16·117	98·9076	+ 13	16·124	49·4720	+ 5	16·130	49·4393	+ 7
16·138	98·9153	+ 13	16·145	49·4760	+ 6	16·152	49·4431	+ 5
17·531	99·4228	+ 11	17·534	49·7291	+ 5	17·537	49·6954	+ 5
18·249	99·6839	+ 5	18·254	49·8603	+ 5	18·261	49·8269	+ 2
18·267	99·6904	+ 5	18·274	49·8636	+ 1	18·279	49·8303	+ 3
18·442	99·7546	+ 9	18·442	49·8944	+ 3	18·442	49·8601	+ 4
18·443	99·7547	+ 6	18·443	49·8944	+ 1	18·443	49·8601	+ 2
19·943	100·3015	+ 8	19·940	50·1675	+ 3	19·938	50·1325	+ 3
19·952	100·3045	+ 5	19·949	50·1690	+ 1	19·945	50·1340	+ 5
19·990	100·3184	+ 5	19·990	50·1765	+ 2	19·990	50·1419	+ 2
19·991	100·3187	+ 5	19·991	50·1767	+ 2	19·992	50·1422	+ 2
20·810	100·6169	+ 2	20·822	50·3281	+ 1	20·834	50·2955	+ 1
20·844	100·6298	− 2	20·855	50·3345	+ 5	20·869	50·3020	+ 2
21·809	100·9812	+ 4	21·810	50·5081	0	21·811	50·4733	− 1
21·812	100·9823	+ 4	21·813	50·5088	+ 1	21·815	50·4741	0
22·829	101·3523	− 3	22·822	50·6923	− 3	22·814	50·6558	− 3
22·853	101·3611	− 2	22·845	50·6965	− 3	22·838	50·6601	− 3
22·993	101·4127	+ 4	22·991	50·7235	+ 1	22·989	50·6880	+ 1
23·000	101·4153	+ 4	22·998	50·7247	0	22·996	50·6893	+ 1
25·333	102·2650	− 2	25·332	51·1501	− 1	25·331	51·1145	0
25·333	102·2651	− 1	25·336	51·1506	− 4	25·335	51·1152	0
25·968	102·4964	− 2	25·972	51·2668	− 1	25·976	51·2318	− 2
25·978	102·5002	0	25·981	51·2686	+ 1	25·986	51·2336	− 2
26·978	102·8647	0	26·985	51·4516	0	26·993	51·4172	0
27·000	102·8727	0	27·007	51·4557	+ 1	27·014	51·4211	0
29·354	103·7297	− 9	29·356	51·8835	− 3	29·358	51·8477	− 3
29·360	103·7321	− 7	29·362	51·8846	− 3	29·363	51·8486	− 3
R.M.S. Error		± 6·7			± 3·2			± 3·3

differences from the values given by the least squares equations. It will be noted that in all three cases the root mean square value of the residuals is equivalent to less than 0·002° Pt.

Since a straight line has been taken to satisfy the relationship between R(Θ) and T° Pt according to the mean of thermometers S.4 and S.5, it follows that Θ has been assumed

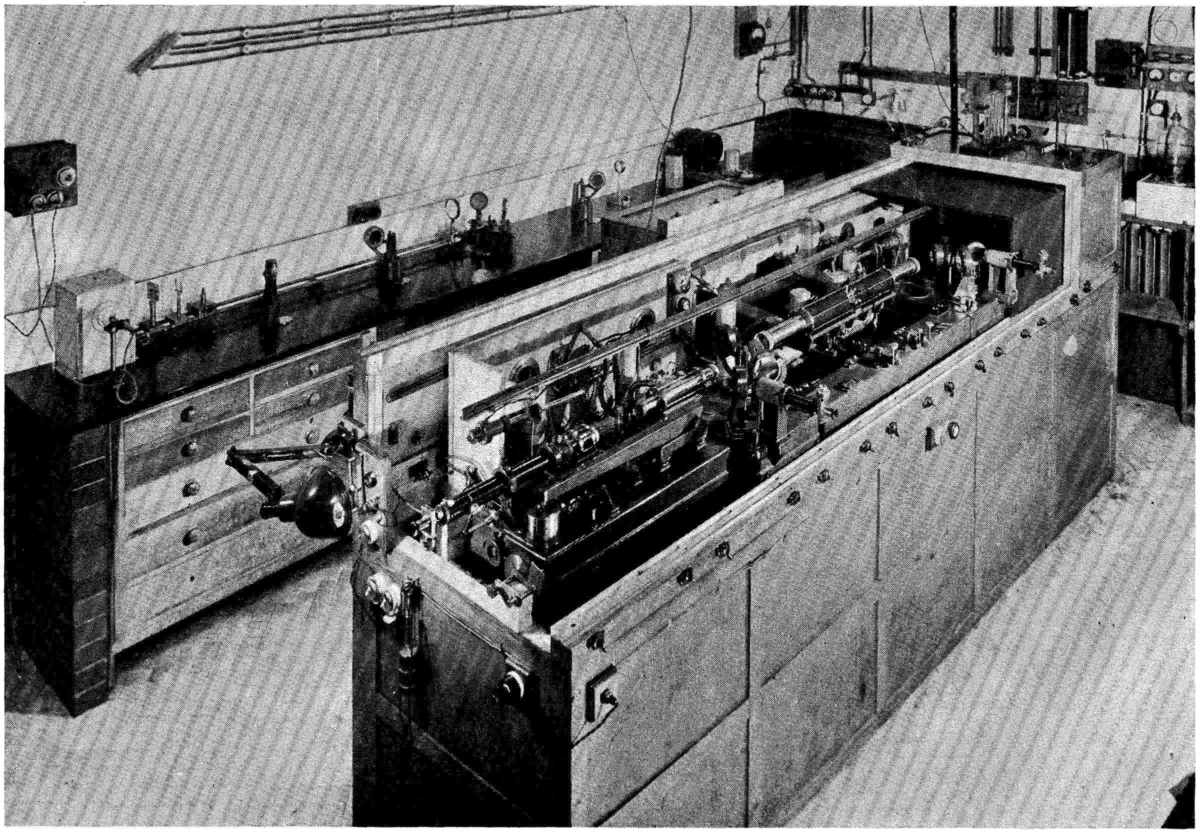

Fig. 21.—General view of Apparatus.

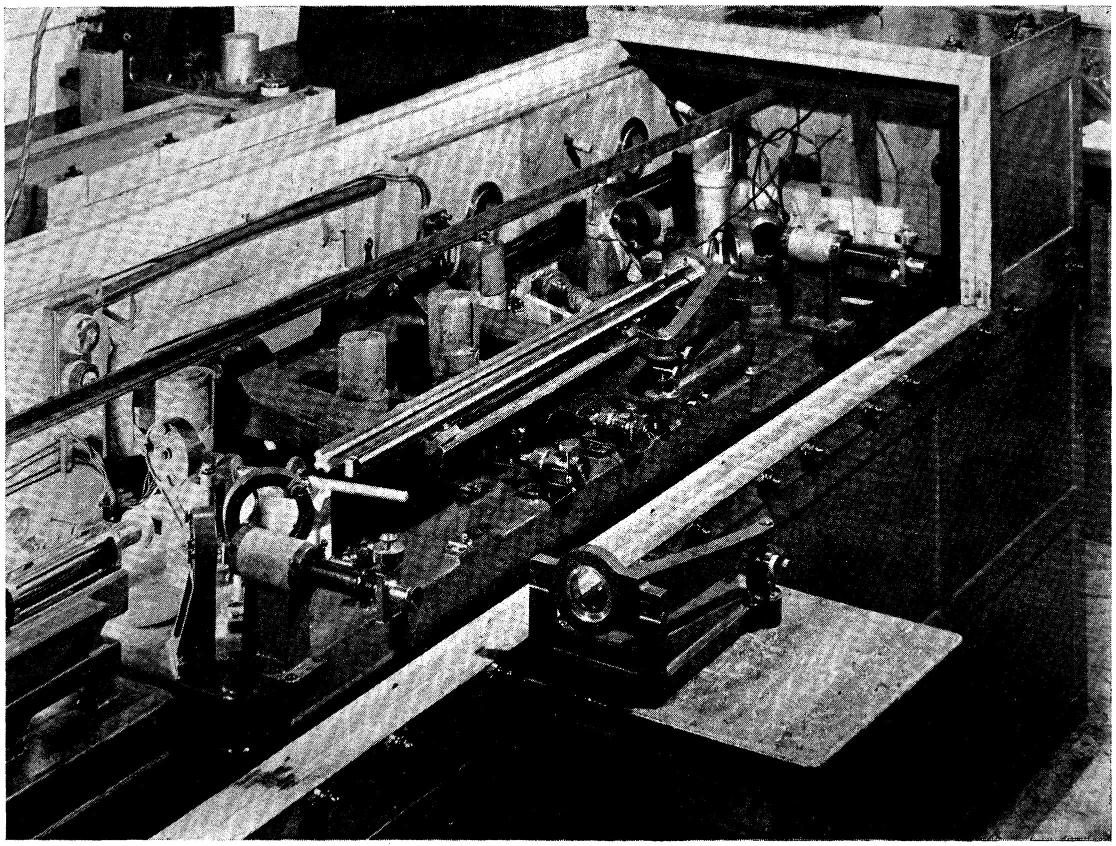

Fig. 22.—The End-gauge comparator.

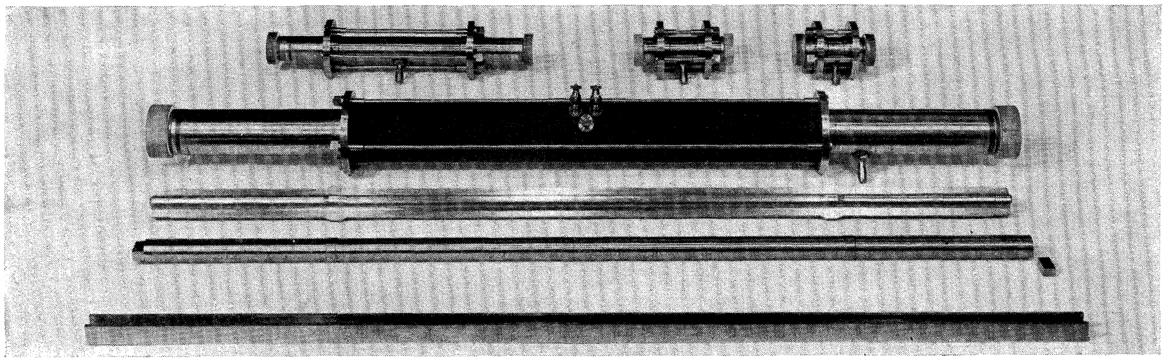

FIG. 23.—Étalons, X-gauge, composite gauge and line-standard.

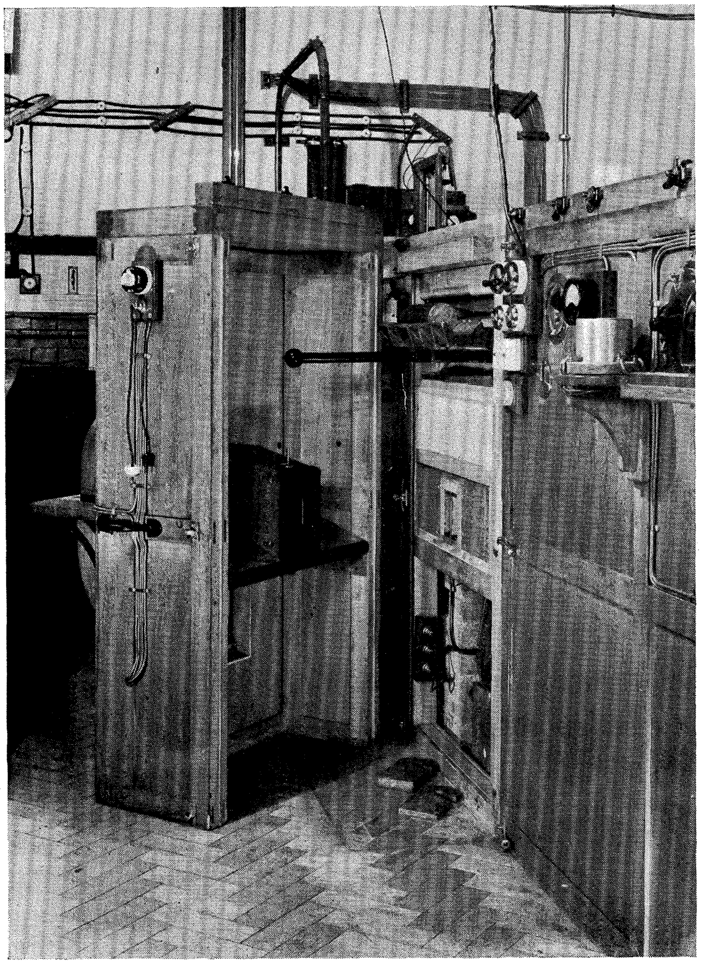

FIG. 24.—Back of enclosure, with cabin open.

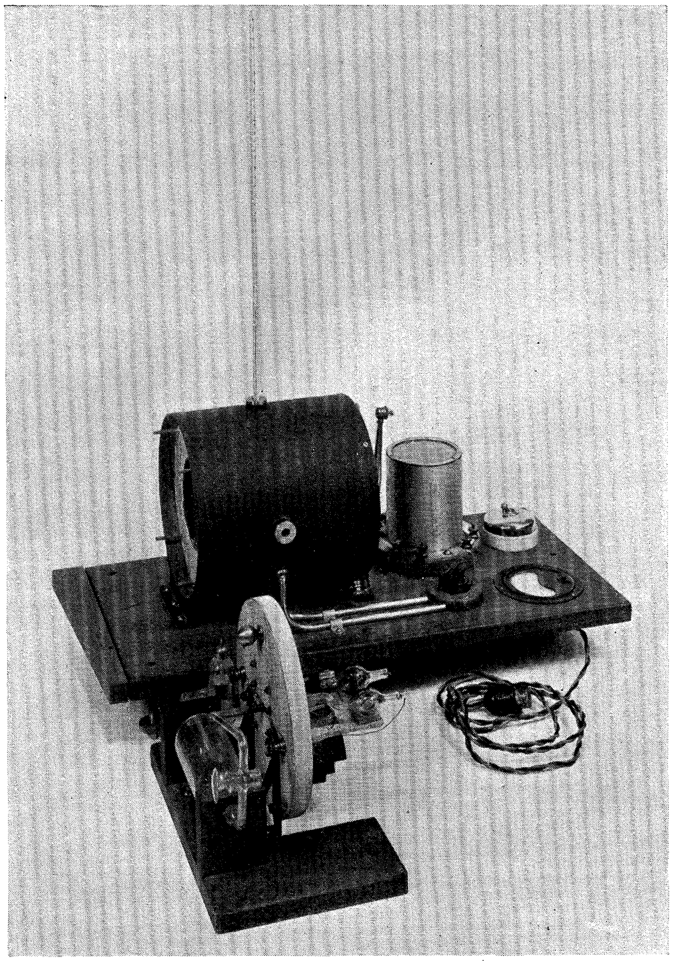

FIG. 25.—Cadmium lamp and furnace.

to have a δ of $1{\cdot}491_5$ (S.4. $= 1{\cdot}493$ and S.5 $= 1{\cdot}490$), a value that is probably very close to the truth. From general experience of recent supplies of platinum from Messrs. Johnson, Matthey & Co., it is very improbable that δ would be outside the limits 1·490 to 1·497, and a change of 0·007 in δ only involves a change of 0·0003° C. in a range of 10° C. in the neighbourhood of 20° C. It will be seen from the tables of residuals that there is some evidence of a slight curvature of the line, but it would evidently not be possible to draw any conclusion regarding δ from observations made to an accuracy of about 0·001° C. over such a restricted range of temperature.

It is evident that equations (1), (2) and (3) should satisfy the condition

$$R(\Theta) = R(\Theta_E) + R(\Theta_W)$$

at all temperatures. When $T = 10°$ Pt the calculated value of $R(\Theta_E) + R(\Theta_W)$ exceeds that of $R(\Theta)$ by 0·00009 ohm (less than 0·0003° Pt) while at 30° Pt the resistances are in agreement to within 0·00001 ohm. A comparison of the values of R_{100}/R_0 calculated for the different sections gives the following results :—

Θ	1·391752
Θ_E	1·391790
Θ_W	1·391704

It would naturally be expected that, since the whole thermometer is constructed from the same length of wire, the coefficients of Θ_E and Θ_W would be identical, but the slight discrepancy found could easily be explained by slight differences in annealing; the temperature of the wire, for example, was certainly not uniform over its length. On the other hand, it is possible that the discrepancy is due to experimental error and on this assumption three " adjusted " laws were computed taking a mean value for the " α " coefficient.

Experience with the étalon mounted in the wave-length apparatus showed that when the independent least square laws were used the difference under steady conditions between Θ_E and Θ_W was quite negligible—not exceeding 0·0002° C. on a mean, and with a maximum for any single observation of 0·002° C. Use of the " adjusted " laws, on the other hand, led to discrepancies of about 0·004° C. It is thus evident that the observed difference between the temperature coefficients of Θ_E and Θ_W has a real existence. It was therefore decided to abandon the adjustment to a uniform value of α. Equations (1), (2) and (3), after arithmetical adjustment to mutual consistency and rounding off to an accuracy more suited to the experimental procedure, become :—

$$R(\Theta) = 93{\cdot}0325 + 0{\cdot}364455\ T \quad (4)$$

$$R(\Theta_E) = 46{\cdot}5319 + 0{\cdot}182309\ T \quad (5)$$

$$R(\Theta_W) = 46{\cdot}5006 + 0{\cdot}182146\ T \quad (6)$$

These equations have been used throughout the present work.

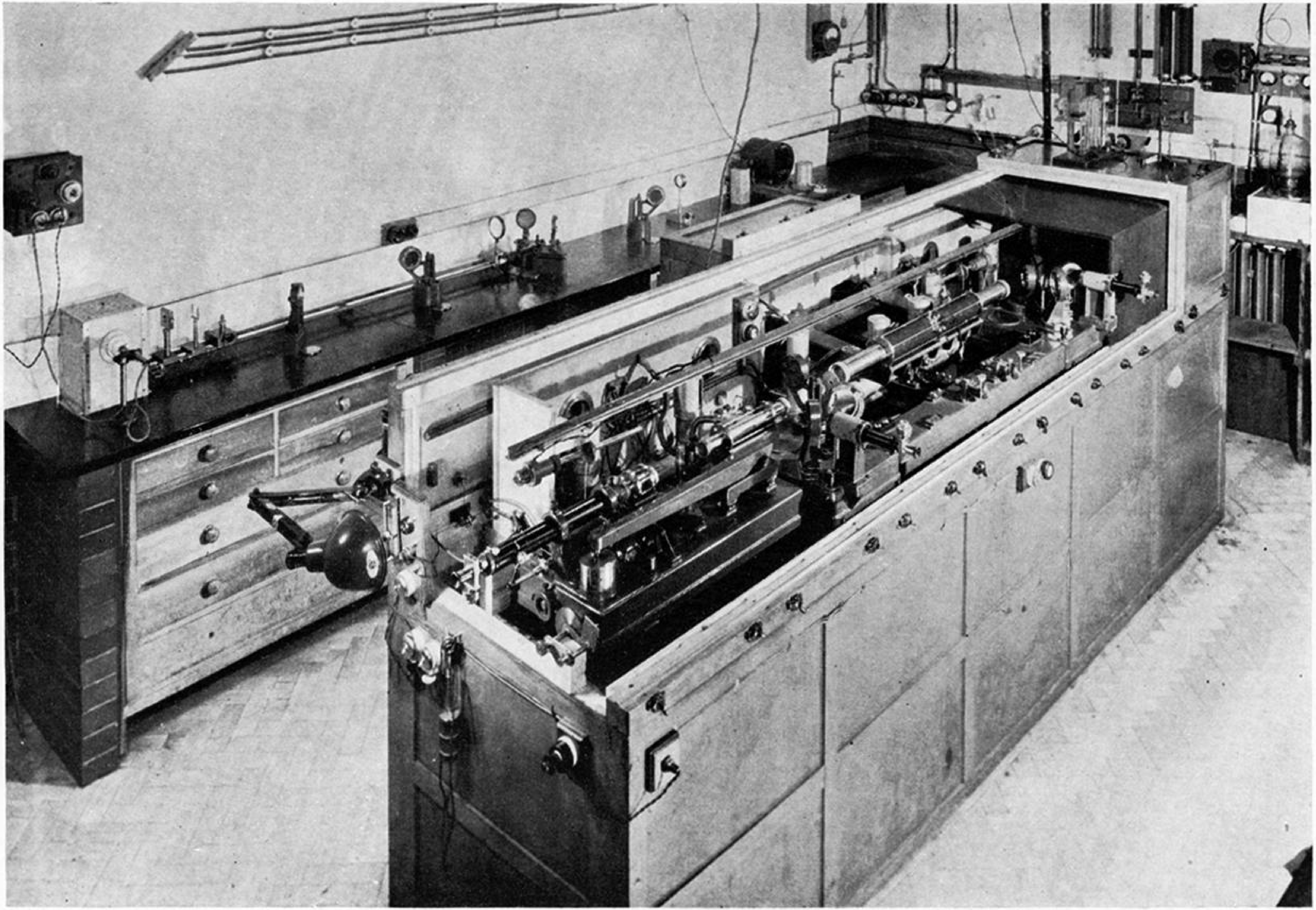

FIG. 21.—General view of Apparatus.

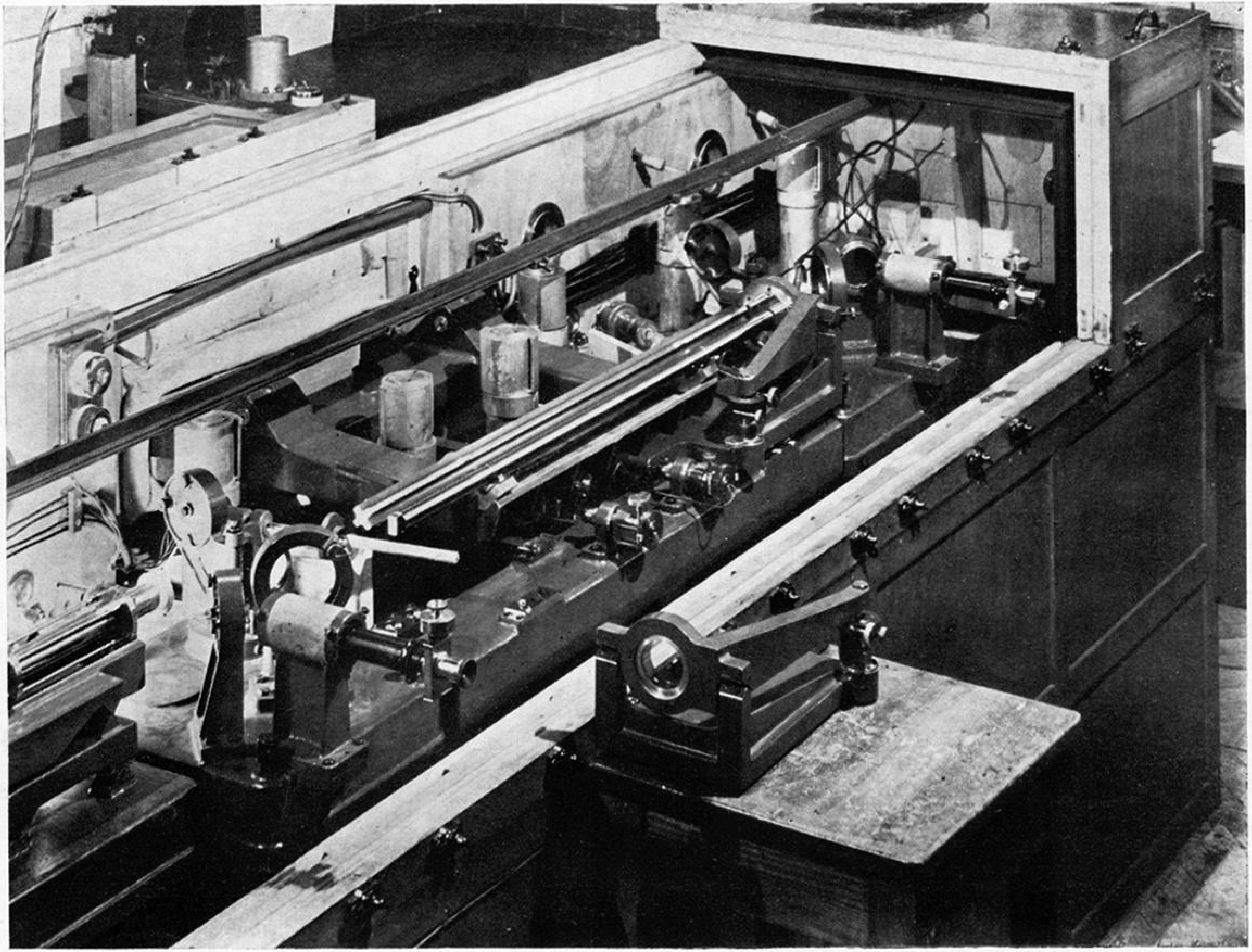

FIG. 22.—The End-gauge comparator.

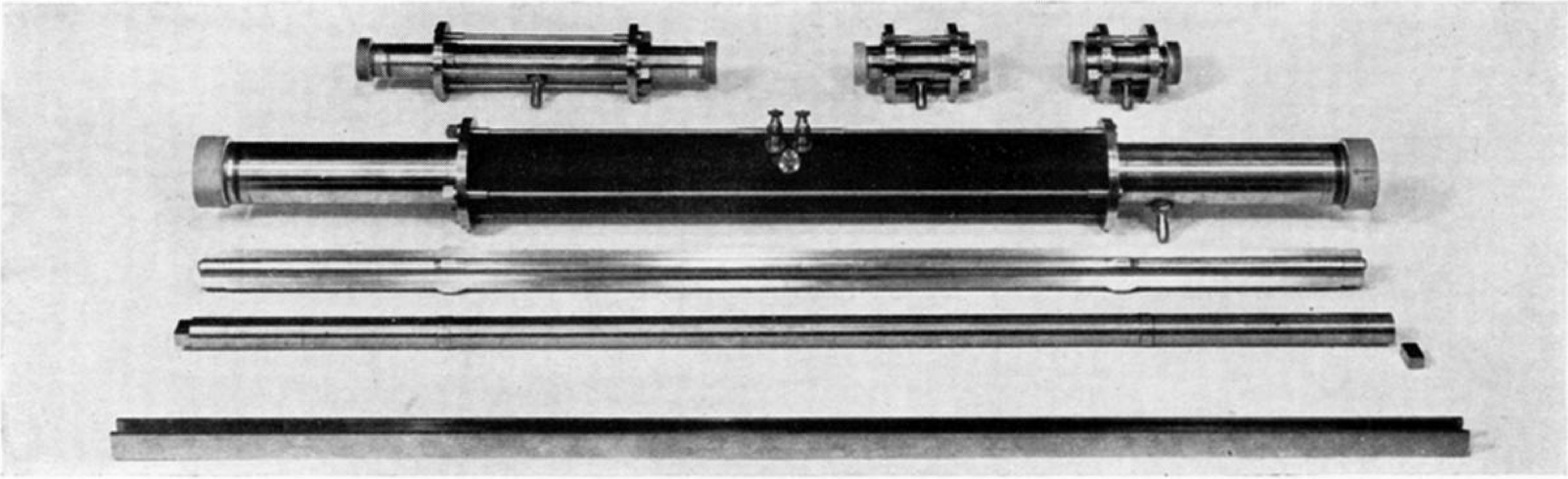

FIG. 23.—Étalons, X-gauge, composite gauge and line-standard.

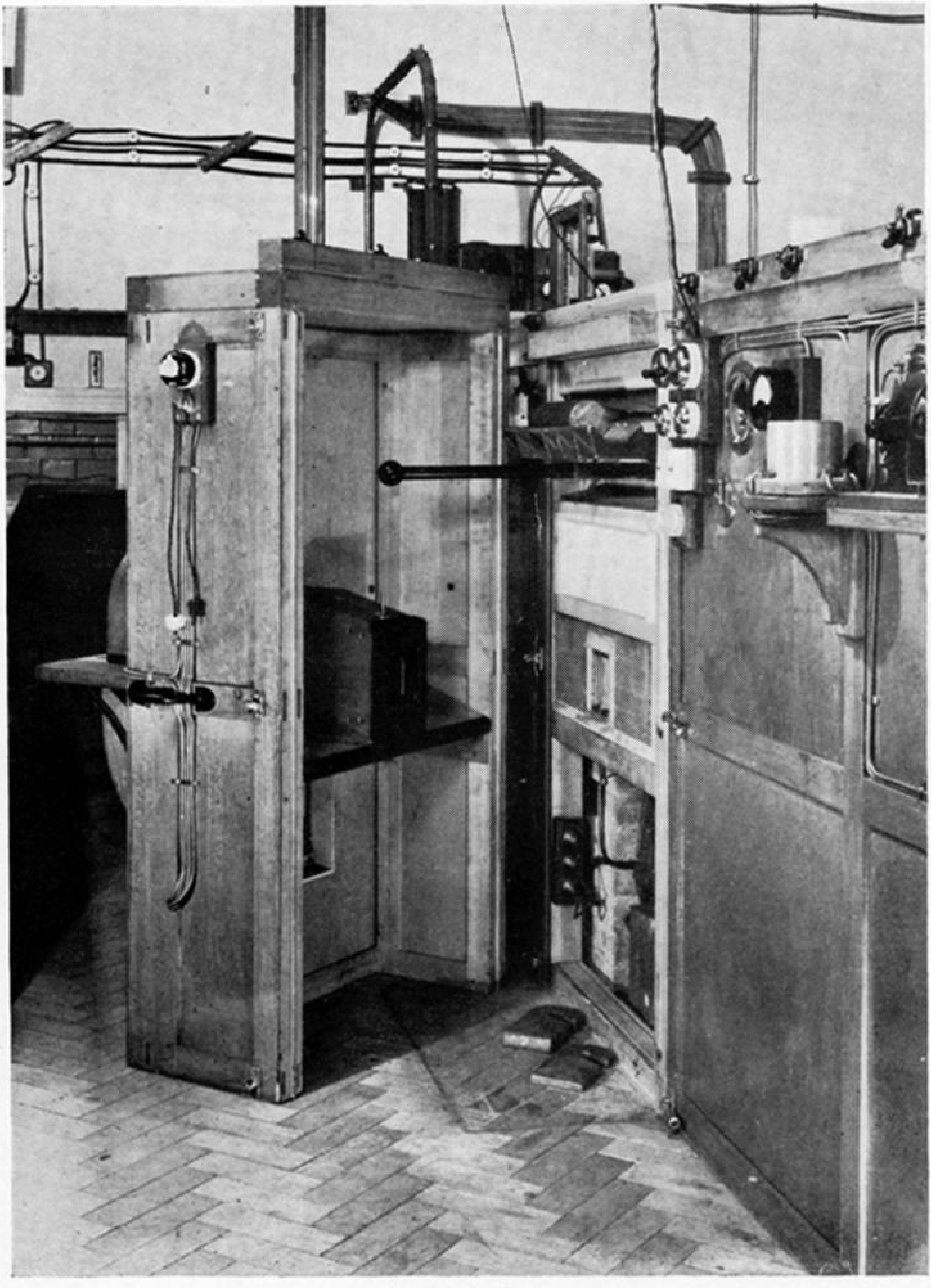

Fig. 24.—Back of enclosure, with cabin open.

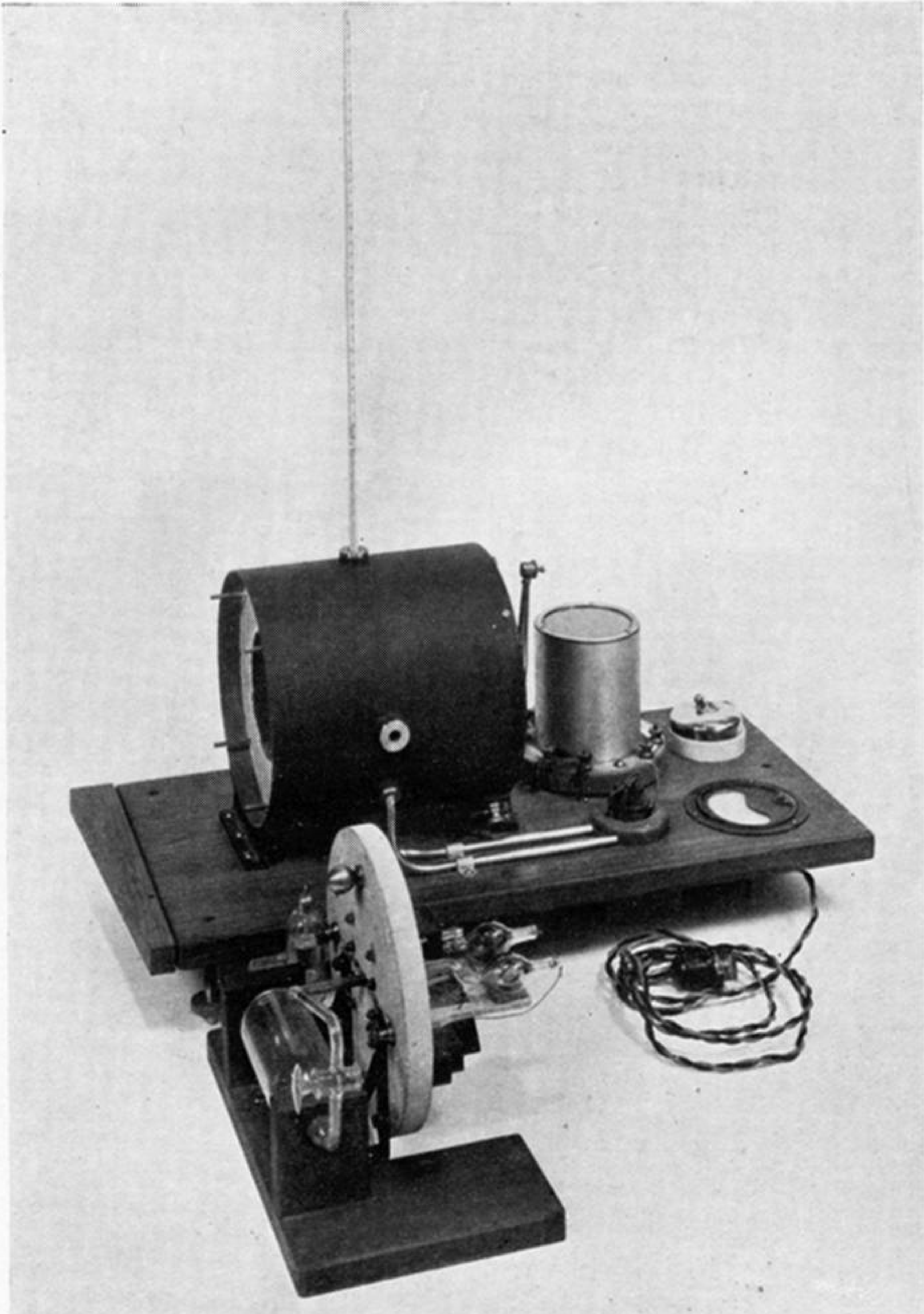

FIG. 25.—Cadmium lamp and furnace.